

LOGICAL PROBABILITY, UNCERTAINTY, INVESTMENT DECISIONS

*Did Keynes's logical theory of probability
have impact on economic thinking?*

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ABSTRACT

The authors of this paper set out to answer the question of whether Keynes's logical theory of probability had an impact on his own economic thinking. The authors review criticisms that had been expressed previously; then, with regard to the applicability of the classical concept of probability to economics, they introduce the foundations of Keynes's logical theory of probability and the difficulties involved in its practical application. This is followed by an examination – within the Keynesian conceptual framework – of the role of uncertainty. The next sections are given over to an analysis of the role of “animal spirits”, and of expectations, with a discussion of investment decisions made from positions of uncertainty. This train of thought focuses on the dilemma of whether there was continuity or a break, over time, in the role of probability in economics within the Keynesian conceptual framework. After this, the authors outline the competing 20th century interpretations of probability embodied by the rearticulated version of relative frequency theory on the one hand, and the evolution of probability theory outside the economics paradigm on the other. The authors conclude with their assertion that probability theory did have a considerable impact on Keynes's thinking on economic theory.

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1 INTRODUCTION

Despite four centuries of scientific research and the widespread acceptance of formalised axiomatic systems, there is no consensus regarding the economic connotations of probability. Instead, various different interpretations have emerged through the ages, but none of these settle the debate regarding what probability actually is writes *Hacking* (1975), expressing his doubts. Under the *classical* approach, probability is regarded as the ratio of favourable instances to all instances. The principle of indifference applies; in other words, if we have no reason to prefer one possibility over another, then they have the same probability (*Laplace*, 1812). The *frequency* approach treats probability as the frequency of successful outcomes in a long chain of trials, and thus the probability of a singular event cannot be determined (*Richard von Mises*, 1928). The *logical* approach sees probability as the logical relationship between the premises of a hypothesis and the supporting evidence (*Keynes*, 1921; *Carnap*, 1959). The *subjective interpretation* relates to the prevailing degree of belief. This value can be determined through a study of our actions (willingness to bet). *Subjectivist* probabilities vary between individuals (*Ramsey*, 1926; *De Finetti*, 1937). The propensity interpretation expresses how the probability of an individual event is a property of the conditions generating the event. In this case, an individual event can also have a probability if it only occurs once (*Popper*, 1997).

The antinomy of a series of events versus a single instance has always been a critical aspect of the role of probability in economics. According to *Arrow* (1951), “*While it may seem hard to give a justification for using probability statements when the event occurs only once, except on the interpretation of probability as degree-of-belief, the contrary position also seems difficult to defend. (...) an extension of this reasoning suggests that in almost any reasonable view of probability theory the probability of a single event must still be the basis of action where there are genuine probabilities*” (*Arrow*, 1951:415).

The interpretations of probability can also be divided epistemologically and ontologically, into inductive and objective versions. The inductive (epistemic) approaches to probability are linked to a person’s knowledge (or belief). In this approach, a probability value describes the degree of knowledge, the degree of rational belief or simply the degree of belief. The theoretical approaches of both *John Maynard Keynes* and *Ludwig von Mises* fall into this category. In contrast, the objective interpretations of probability treat probability as a property of the objective, material world, which has no relationship whatsoever to human knowledge or belief. *Richard von Mises’s* objective probability approach – the frequency interpretation – falls into this category; his fundamental objective was to make

probability theory analogous to other (exact) sciences (*Backhouse–Bateman*, 2006).

Keynes declared the following his *A Treatise on Probability* (TP) published in 1921, which laid the foundations for his subsequent views:

“There appear to be four alternatives:1) Either in some cases there is no probability at all; or 2) probabilities do not all belong to a single set of magnitudes measurable in terms of a common unit; or 3) these measures always exist, but in many cases are, and must remain, unknown; or 4) probabilities do belong to such a set and their measures are capable of being determined by us, although we are not always able so to determine them in practice” (Keynes, TP:33).

Keynes’s 1936 *The General Theory of Employment, Interest and Money* (GT) can be analysed together with his essay *The General Theory of Employment* (GTE) published in 1937 in the *Quarterly Journal of Economics*. In the GT, Keynes broke away from the partial equilibrium analysis-based approach of classical economics, and in the main his theory can be regarded as an aggregated general equilibrium framework centred on uncertainty. Keynes wrote the GTE with the purpose of summarising what the GT had to say and putting forward an even more convincing argument for its claims. In these latter two papers, Keynes expresses his view that the performance of the economy as a whole is mainly determined by the volume of investment. Keynes considered the quantity of investment to be the factor defining “the level of output and employment as a whole” (Keynes, 1937:221).

Given that the basis for Keynes’s conceptual framework is the assumption of uncertainty, *Coddington* (1982) is justified in asking whether or not certainty is attainable. Keynes interprets certainty (rational belief) as something that requires not only complete confidence in the belief, but also the accuracy of the belief. In Keynes’s case, this certainty equates to knowledge. Keynes, therefore, is not as sceptical and agnostic as he is assumed to be. Keynes distinguishes between two types of knowledge: the kind of knowledge that can be directly obtained, and that which can only be obtained indirectly – one is the directly knowable part of rational belief, and the other is what we can deduce through argument. (Keynes, 1921:12).

Keynes committed himself to the broader logic of conclusiveness rather than simple logical deduction and numerical probabilities. While the relative and absolute nature of probabilities both suggest that they do not necessarily exist as a part of material reality, an empirical propensity to understand reality is nevertheless possible. *Lawson* (1988) describes Keynes’s attitude towards this as follows:

“...throughout his total contributions he is explicit that ... a priori thought is considered always to be open to constant modification and correction through continual interaction with experiences of the real world” (Lawson, 1988:56).

Keynes never left any doubt that probability statements, not being regarded as relative frequency, should be contingent on the current evidence and knowledge, and that changes in them should also be regarded as natural. Moreover, the attention paid by Keynes to the fundamentally qualitative nature of reality suggests that informal argument and intuitive judgement are both necessary for economic reasoning, as a formalised model and statistical conclusiveness, and this shows a commitment to deductive logic.

These thoughts are expressed concisely by *O'Donnell* (1999:93) in his description of Keynes's probability theory: "there is a consensus with regard to empiricism that experience is a prerequisite for knowledge; Keynes's theory of knowledge goes further than this, maintaining that much knowledge is impossible without a priori reasoning or intuition". *Keynes's theory of probability had to emphasise intuition, and his view in this regard did not weaken, but in fact strengthened, which suggests that Keynes was a rationalist.* The same can be said of his predictions regarding future value, because Keynes also exercised rational judgement in recognising that rational statements about the future are so uncertain that they cannot serve as the basis for rational action.

2 CRITICISM OF THE CLASSICAL PROBABILITY CONCEPT

The biggest problem when examining the role of probability is that there is no explicit and comprehensive definition of probability that could be applied universally to all branches of science. This study does not deal with the axioms, postulations and paradigms of mathematical probability calculus. This paper deals with the aspects of probability that are related to economics questions in general, and specifically to investment decisions. Accordingly, based on the approach probability may be objective, subjective and logical by nature, and based on the method it may be classical probability, relative frequency and propensity interpretation. Probability can be interpreted in respect of objective reality, based on the relationship between the individual and reality, and with regard to the degree of the individual's knowledge.

The oldest of the special probability interpretations is the classical approach. In essence, the classical approach means that the probability of an event, in a given random trial, is the ratio between the equal-chance outcomes related to a given event and the number of equal-probability outcomes. The substance of classical probability was most fully described by Laplace (1812). He derived probability from general determinism when he wrote:

"We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at a certain moment would know all

forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes” (Laplace, 1812:4).

Examining the source of the classical interpretation of probability, Szabó (2011) concludes that neither the past nor the future holds any uncertainty for Laplace’s all-knowing demon; for us mortals, however, this ultimate knowledge is unattainable.³ On this basis, probability originates from the limitations of human knowledge. To substantiate this premise – over time – three essentially identical principles have been articulated by the great thinkers on the problematics of probability. According to the “law of sufficient reason” the symmetry of outcomes presupposes identical probability for each outcome. Based on the “law of insufficient reason”, if we do not know which outcome is more likely, then we assign the same probability to each one (Laplace, 1812; Bernoulli, 1713). The “principle of indifference” states that equal probabilities must be assigned to each of several arguments if there is an absence of positive ground for assigning unequal ones” (Keynes, 1921:45).

Keynes’s definition is as follows:

“The Principle of Indifference asserts that if there is no known reason for predicating of our subject one rather than another of several alternatives, then relatively to such knowledge the assertions of each of these alternatives have an equal probability” (Keynes, 1921:42).

Butos–Koppl (1995) points out that this also means the principle only works if there is a sound basis for assigning a special set of non-identical weights. In the absence of a basis for the assignment of a special set of non-identical weights, the principle of indifference demands the assignment of identical weights. Keynes ar-

3 Laplace writes the following on the causes of unattainability of knowledge: “All events, even those which on account of their insignificance do not seem to follow the great laws of nature, are a result of it just as necessarily as the revolutions of the sun. In ignorance of the ties which unite such events to the entire system of the universe, they have been made to depend upon final causes or upon hazard, according as they occur and are repeated with regularity, or appear without regard to order; but these imaginary causes have gradually receded with the widening bounds of knowledge and disappear entirely before sound philosophy, which sees in them only the expression of our ignorance of the true causes” (LAPLACE, 1812:3).

gued that “it is a necessary condition for the application of the principle, that [we take as our basis] indivisible alternatives of the form” (Keynes, 1921:65)⁴.

Laplace is in no doubt that probability relates partly to our knowledge and partly to our lack thereof. According to him, the principle of insufficient reason states that if we have no cause to believe more in the occurrence of one out of two or more events than in the occurrence of another, then the events must be considered to have equal probability.

Szabó (2011) highlights that, since Laplace derives his concept of cognitive probability from general determinism, “we may regard the present state of the universe as the effect of its past and the cause of its future” (cited above); in other words, the world is governed by determinism. We mortal souls, “however, do not understand the threads that tie such events to the whole system of the Universe (and thus) make them dependent on aims and randomness” – in other words, probability is merely epistemic in nature. If indeterminism reigns in the world; that is, if the following states of the world do not unambiguously record each other, then objective probability could be some kind of degree of that indefiniteness; that is, the metaphysical quantity that in some way defines the “distribution” of physically possible future states.

The classical concept of probability emerged as a formalised theory in the second half of the 19th century as the *theory of relative frequency*. Its main proponent was *John Venn* (1888–1962), who regarded *the sequence and the limit* as the cornerstones of his theory. Observations of games of chance showed that with an increase the number of experiments (in a sufficiently long series of results) the relative frequency fluctuates around a defined value, “holding” to a certain value. This is the limit of the sequence, which is regarded as the probability of the event. Venn created the framework on which the frequency interpretation could be based. He defined the concept of the “sequence”, which has central importance in frequency theory, and is a chain of events, each having certain important properties. Probability is related to an infinite sequence of recurring events.

The sequence of events is a prerequisite for probability expressed as relative frequency. The attempt to determine the probability of occurrence of a given event (to assign a mathematical probability value to its occurrence) presupposes the existence of a sequence of which the given event is a part. *Proponents of the concept of relative frequency identified probability with statistical frequency.*

4 “In short, the Principle of Indifference is not applicable to a pair of alternatives, if we know that either of them is capable of being further split up into a pair of possible but incompatible alternatives of the same form as the original pair” (KEYNES, 1921:67).

Amsterdamski (1965) takes a mostly sympathetic approach to Venn's theory of relative frequency. He believes that the frequency theory captures the fact that, in everyday probability statements one talks about how much "chance" there is of a certain type of event occurring, and not about the extent to which any possible data logically support the appropriate hypothesis. In his opinion, Venn believes that probability only relates to *mass events* that can be statistically captured. Amsterdamski supports the relevance of the frequency theory with numerous everyday examples. In his view, certain statistical distributions are very stable: These include the ability of people to live to a certain age, the distribution of newborn babies by sex, the results of throwing dice or spinning a roulette wheel, the incidence of persons with specified distinguishing features within certain biological populations, the distributions relating to the decay of radioactive atoms. The actions of insurance companies, gamblers or physicists making statements on the future state of a microsystem are based on the conviction that the concept of probability relates to physical reality, and not to the logical relationships between judgements (Amsterdamski, 1965:268).

There were widespread doubts about the applicability of relative frequency theory to fields of economics, and the theory was subjected to harsh criticism. The definition of probability as a limiting value of relative frequencies in an infinite series assumes that the number of experiments continues beyond every limit towards infinity. Conducting an infinite number of experiences, however, is impossible for two reasons. Firstly, the human lifespan is finite; and secondly, the circumstances of the events constituting the series can also change over a long period of time. Another objection is related to the concept of probability itself. The "favourable outcome/all outcomes" ratios do not make up a convergent series even if the relative frequencies fluctuate around a determined value. *In other words, the best that can be said is that relative frequency is a good approximation of the probability value.*

Arrow (1951) takes a highly critical view of the identification of relative frequency with probability. Firstly, he criticises the reduction of probability to a simple ratio, as follows:

"In the whole calculus of probabilities, there is a process of evaluating the probabilities of complex events on the basis of a knowledge of the probabilities of simpler ones. This process cannot go on indefinitely; there must be a beginning somewhere. Hence, in the study of games of chance, an a priori judgment is usually made as to certain probabilities. But in the usual types of events which occur in insurance or business affairs, there is no natural way of making these judgments; instead, the appeal is to past observations, if anything" (Arrow, 1951:411).

Arrow questions the equivalence of relative frequency and probability in relation to the law of large numbers. In its simplest form, this law articulated by J. Bernoulli states that in a series of independent trials where the given event E may occur with a constant probability p in each experiment, by selecting a sufficiently large number of trials it is possible to make the probability of the relative frequency of occurrence of E in n trials differ from p by more than an assigned positive value. Naturally, it remains true that given an infinite number of trials, however large the number of trials, “*we cannot identify relative frequency with probability itself*” (Arrow, 1951:414.).

Arrow (1951) reiterates Laplace’s (1812) well-known assertion quoted above, albeit in a different thought structure. Arrow justifiably asks “whether or not there is any ‘objective’ uncertainty in the economic universe, in the sense that a supremely intelligent mind knowing completely all the available data could know the future with certainty” (op. cit. 405). His answer to this question is that “the tangled web of the problem of human free will does not really have to be unravelled for our purpose; surely, in any case, our ignorance of the world is so much greater than the ‘true’ limits to possible knowledge” (op. cit. 406).

On this basis, *we are left in no doubt that Arrow does not identify relative frequency with probability* and that he sees the latter fundamentally as an epistemic rather than an ontological problem, believing that probabilities are only rarely known with certainty. In a strict interpretation, only idealised schematic instances of known probability are unambiguous, such as a dice roll or a coin toss, the rules of which are beyond doubt. In real-life situations – even if we act on the basis of a firm probability estimate – we cannot be certain that this estimate is wholly accurate, because there is also uncertainty.

The critics of classical probability theory believed that frequency probability does not encompass everything that we think of as probability. In his comparative study of schools of thought on probability, Hauwe (2011) concludes that “*clearly the random frequency definition of probability is too narrow to encompass what we mean when we use the term probability. We do say of unique events that they are more or less probable. Many decisions that people make daily are based on probability statements that have no frequency interpretation*” (Hauwe, 2011:500).

Knight (1921) questions the wide-ranging applicability of frequency probability based on the uniqueness of economic decisions when he writes:

“The ... mathematical, or a priori type of probability is practically never met with in business. ... Business decisions ... deal with situations which are far too unique, generally speaking, for any sort of statistical tabulation to have any value for guides. The conception of an objectively measurable probability of chance is simply inapplicable” (1921:219).

However, Knight also commented that, besides the inapplicability of probability to decision-making, another important circumstance is that “at the bottom of the uncertainty problem in economic is the forward-looking character of the economic process itself” (op. cit. 237).

By the end of the 19th century it had become clear that the classical interpretation of probability does not guarantee the quantification of probability, and nor is it suitable for the probability rating of individual events and decisions in the absence of a series of events. These limitations stimulated intellectual exploration and the development of new probability interpretations.⁵

3 THE KEYNESIAN LOGICAL THEORY OF PROBABILITY

In his 1921 work based on logic and philosophy, *A Treatise on Probability*,⁶ Keynes elaborated a conception of probability that placed the roles of uncertainty, expectations and behaviour in decision-making on a radically new footing. Keynes defined one of the work’s declared aims as being that it “theorises the methods of reasoning that we actually use, as opposed to the ultra-rationality of perfect logical insight that is held to be omniscient” (Keynes, 1921:135).

Another paper by Keynes (1937) contains an explanation for his departure from the fundamental ideas of classical economics:

“I sum up, therefore, the main grounds of my departure [from the traditional theory] as follows: The orthodox theory assumes that we have a knowledge of the future of a kind quite different from that which we actually possess. This false rationalism follows the lines of the Benthamite calculus. The hypothesis of a calculable future leads to a wrong interpretation of the principles of behaviour which the need for action compels us to adopt, and to an underesti-

5 WEINTRAUB (1975) concisely expressed the situation prevailing at the turn of the 19th and 20th centuries as follows: “At that time the only explicit theory which delineated the meaning of the proposition “the probability that x is y is p ” was that of Venn, which provided a relative frequency interpretation of probability statements. Such a theory asserted that the meaning of “the probability that x is y is p ” was that a large number of cases had been examined in which x was y and x was not y , and p was the proportion of the former in the total number of cases” (1975:532).

6 “Between 1906 and 1911 Keynes was devoting all his spare time to the theory of Probability. ... In 1912 other work supervened, and his treatise had to be left on one side until 1920, when he polished it up before its appearance in 1921. Thus, it was his main work from the age of twenty-three to twenty-nine.” This work attempted to carry out, for the theory of probability, the program initiated by *Russell* and *Whitehead* for mathematics, namely, to provide a logical foundation for the subject” (WEINTRAUB, 1975: 535).

mation of the concealed factors of utter doubt, precariousness, hope and fear”
(Keynes, 1937:221).

Here, Keynes is claiming that the classical (traditional) theory encompasses situations that are handled with the tools of probability in keeping with the application of risk. The classical theory assumes that a person can maximise the expected payouts despite that fact that the likely values cannot be reliably calculated. Moreover, the individual must act today, whereas the impacts of his or her choices will only become known in the future; however, every economic action taking place at a certain time has intertemporal consequences. An economic entity has to base its decisions on something; this thing may be a processing of the recent past or something else, although such a framework for decision-making “being based on so flimsy a foundation ... is subject to sudden and violent changes” (op. cit. 214).

Keynes did not believe that entrepreneurs make a list of all the possible future outcomes, assign a probability to every item on the list, and then calculate the expected value. Entrepreneurs cannot perform a Benthamite calculation (Bentham, 1789) of long-term values. Keynes stated that “our existing knowledge does not provide a sufficient basis for a calculated mathematical expectation” (Keynes, 1936:152).

The foundation of Keynes’s structure is differentiation between any two items of the probability assignment, the premise. In his view, every argument h originates from the conclusion derived from a set of premises and is based on the logical probability relation between a and h . Typically, the premises only offer partial assistance in reaching the conclusion. Assuming that the premises are true, it would not be rational to believe in the conclusion with complete confidence; it would be more rational to believe it with a certain degree of confidence. O’Donnell (1990a) highlights that Keynes’s probabilities thereby express several aspects of the argument – they show the degree of partial entailment (that is, the extent to which a follows from h ; they express the degree of rational belief, how much a can be believed in a knowledge of h); and they also express the degree of certainty: Keynes denoted this with the symbol a/h .⁷

Keynes regarded his probability theory, like economics, to be a part of logic; and at the beginning of his treatise he made it clear that his theory – in essence –

7 KEYNES’s theory of probability in *A Treatise on Probability* (TP, 1921) is based on Boole’s mathematical logic as set out in Boole’s work *The Laws of Thought* (LT, 1854). Based on this, reality is relational by nature. Probability is linked to the arguments, not to the outcomes, events or individual statements. The arguments comprise two types of statements. These two types of statements are referred to as premises and conclusions. At the same time, statements can relate to outcomes and/or events. Keynes denotes the premise with an h and the conclusion with an a . The equation $a/h = \alpha$ is Keynes’s original symbol of the probability relation between a and h (BRADY, 2018).

was objective. For him, probability of the *degree of rational belief*, not simply the degree of belief. It is worth quoting the relevant passage – as a summation of Keynes’s probability doctrine – in its entirety:

“The terms certain and probable describe the various degrees of rational belief about a proposition which different amounts of knowledge authorise us to entertain. All propositions are true or false, but the knowledge we have of them depends on our circumstances; and while it is often convenient to speak of propositions as certain or probable, this expresses strictly a relationship in which they stand to a corpus of knowledge, actual or hypothetical, and not a characteristic of the propositions in themselves. A proposition is capable at the same time of varying degrees of this relationship, depending upon the knowledge to which it is related, so that it is without significance to call a proposition probable unless we specify the knowledge to which we are relating it. To this extent, therefore, probability may be called subjective. But in the sense important to logic, probability is not subjective. It is not, that is to say, subject to human caprice. A proposition is not probable because we think it so. When once the facts are given which determine our knowledge, what is probable or improbable in these circumstances has been fixed objectively, and is independent of our opinion. The Theory of Probability is logical, therefore, because it is concerned with the degree of belief which it is rational to entertain in given conditions, and not merely with the actual beliefs of particular individuals, which may or may not be rational” (Keynes, 1921:3–4).

Keynes (1921), in his *Treatise on Probability*, rejected the theory of relative frequency. Instead, in its place, Keynes proposes that probability is not related to the balance of favourable and unfavourable evidence, but to the balance of the absolute quantity of relevant knowledge and that of relevant ignorance, in such a way that the discovery of new evidence increases the weight of the argument. In Weintraub’s (1975) view, at that time Keynes’s argument consisted of the following: “In order for probability to use probability to guide choice *in matters of fundamental uncertainty one needed to discuss not only the probability but also the confidence one held in that probability*. ... Consequently, an economic agent ought not maximise expected payoffs, when each of an array of payoffs is assigned a probability number by the agent, if he has little confidence in those probabilities”.

O’Donnell made an important discovery in recognising that the TP was more logical than epistemic in character. Keynes’s fundamental aim was to solve the conundrum relating to the rational, but not conclusive argument; to analyse and confirm those non-quantifiable arguments, in science, everyday life and elsewhere, that can be believed to be rational in a certain sense, but which does not have deductive evidentiary force. Keynes’s solution was to place this family of arguments under the rule of logic by making probability theory synonymous with

logic theory. *Thus, probability became a general theory of logic applied to the logical relationship between any pair of arguments, with the inclusion of traditional deductive logic as a special case.* The natural driver of this project was the logical concept of probability, in which probability related to the logical relationship between statements; a typical example of this was the argument in which the premises only partially support the conclusion. Keynes referred to these relationships of partial support or attraction – between premises – as probability relationships, and he came to the additional conclusion that these relationships express the degree of rational belief, which guaranteed for individuals the ability to draw conclusions from such arguments.⁸

Keynes's TP is concerned with the path, leading from the premises to the conclusion, that is conceivable but not certain. Starting out from the premises, we attempt to confirm a certain degree of the rational belief for every variant of the conclusions. We can do this by assuming a certain logical relationship between the premises and the conclusions. The version of rational belief that we arrive at in this way can be designated as probable (or bordering on the certain), and the logical relationships that we gain with this perception can be marked as probability relationships. (cf. Hauwe, 2011). *Downward* (1998) argues that “*from a purely logical point of view new evidence implies a new, unique, probability relation. Probability is not something that can be learned about but is a logical relationship, between sets of propositions, expressed as a conditional statement in the light of background knowledge or evidence. Representing probabilities with reference to a relative frequency distribution thus cannot make sense.*”

One of the most disputed aspects of Keynes's logical theory of probability is its objective or subjective nature. *Rosser* (2001) asserts that an important aspect of Keynes's view on probability is that he himself considered them to be essentially subjective; that is, something that can be constructed on the basis of internal logic rather than from mathematical calculations of the distribution of external observations. Our earlier Keynes quotes, and their interpretation by critics and sup-

8 HÁRSING (1971) emphasises that, in Keynes's approach, probability is a peculiar logical relationship: the relationship between premises and the conclusion. It is customary in literature on scientific theory and logical probability to refer to the premises as evidence. This designation is fitting in two ways: (1) The premises make up the knowledge that we accept as true within a given train of thought; that is, we consider them to be evident. (2) The primary meaning of the word “evidence” is similar to that of “proof”. The methodological function of the premises is also to lend probability to knowledge (conclusions), the truth of which we cannot recognise directly, but only indirectly through statements in a logical relationship determined by them. In terms of their origin, such statements are hypotheses. This latter interpretation of “evidence” must certainly have contributed to the definition of probability as the degree to which hypotheses are proven.

porters alike, prove neither unilateral objectivity nor unconditional subjectivity in relation to Keynes's theory.

Hársing (1965) provides a convincing explanation to resolve this dilemma. His analysis starts out from the fact that we can differentiate between objective phenomena that exist independently of human consciousness, which are customarily referred to as events in probability theory, and the subjective mirror-images of these that are created in our consciousness. In this way, we can describe the extent of the basis for an objective phenomenon as objective probability, and the extent of some inferred knowledge (hypothesis) that has been determined indirectly (based on observations, experiments etc.) as logical probability. Essentially, the latter form of probability is also objective in nature, because the probability of a given hypothesis being correct is based on knowledge (judgements) where the phenomena encapsulated by the judgements are objectively related to the phenomenon in the hypothesis. *Just as logical probability directly describes the relationships between judgements, and since the judgements – like all human knowledge – are subjective images of the phenomena of objective reality, logical probability also has a subjective side.* Hársing's recognition of the subjective aspect of logical probability has great importance because it resolves the main dilemma of the Keynesian logical probability theory with regard to subjectivism. Hársing perceptively concludes that the "accusation" of subjectivism levelled at Keynes is not based on the fact that Keynes defines logical probability as the degree of rational belief. He believes that the expression "degree of rational belief" is misleading, and although it creates the impression of subjectivism, in reality the substance of this concept, for Keynes, is objective: the degree to which the hypotheses are founded. For him, subjectivism stems from the fact that he regards the concept of logical probability as exclusive, and also extends it to the description of objective phenomena. Ultimately, this leads to a rejection of probabilities that are totally independent of human consciousness, and to the subjectivisation of objective phenomena (Hársing, 1965:951).

The use of mathematical probability calculus presupposes the measurability of probabilities. In his treatise, Keynes (1921) denied that all probabilities are numerically measurable or suitable for positioning on a standardised scale of sizes. In his later work, Keynes (1937) claims that probabilities associated with the relatively distant future are not measurable; he mentions that things like "the prospect of a European war" or "the rate of interest twenty years hence", are so uncertain that "there is no scientific basis on which to form any calculable probability whatever. We simply do not know." (Keynes, 1937:213–214). The probability of events that influence the growth of capital cannot be measured either. Accordingly, nor can the present value of current investments be reliably calculated.

Kay (2012) concurs with Skidelsky, who believed that understanding Keynes's approach to probability is the key to understanding the rest of his work. Keynes believed the financial and business environment to be characterised by "radical uncertainty". The only credible answer to the question of "what will interest rates be in twenty years' time" is "we simply don't know"⁹.

Hársing (1971b) emphatically points out that Keynes does not limit probability calculation to the analysis of games of chance and insurance transactions, and even if it means partially relinquishing the quantitative aspect, he attempts to retain the original broadness of the concept of probability. (It should be noted that probability calculus, as a mathematical theory, emerged as a result of the work of B. Pascal és Jacob Bernoulli in relation to the analysis of the outcomes of games of chance) (Hársing, op. cit. 242).

It is generally accepted that Keynes's concept of logical probability is "of a comparative nature". This necessarily follows from the effort to elaborate a logical theory of probability that was more exact than before, without narrowing the definition of probability. Keynes resisted excluding, from the theory, probabilities that did not lend themselves to quantitative evaluation. He wrote the following on this:

"I maintain that there are some pairs of probabilities between the members of which no comparison of magnitude is possible; that we can say, nevertheless, of some pairs of relations of probability that the one is greater and the other less, although it is not possible to measure the difference between them; and that in a very special type of case ... a meaning can be given to a numerical comparison of magnitude" (Keynes, 1921:34).

Keynes generally uses his concept of probability in the comparative sense, but he does not rule out the possibility of a quantitative interpretation in special cases.¹⁰

Brady (1983:27) points out that Keynes does not oppose the attempt to approach probability with an estimate that is subject to lower or higher barriers or limits. This argument is related to the following quote, taken from Keynes:

9 KAY (2012) believes that this was forward-looking and prescient commentary on the part of Keynes. Twenty years before publication of the TP we find ourselves in 1941, when Great Britain, at a critical stage of the Second World War, is engaged in a life-and-death struggle for survival. Keynes saw the future more clearly than most, but when it came to what specific events would take place, he simply did not know. Like everyone else.

10 HÁRSING (1971) believes that Keynes clearly saw the paradox of scientific theory whereby an increase in the exactness of definitions (the "arming" of definitions) usually leads to a narrowing of their scope. Based on examples from court proceedings and betting, he reaches the conclusion that in most cases the concept of probability can only be used in a comparative sense.

“It is evident that the cases in which exact numerical measurement is possible are a very limited class ... The sphere of inexact numerical comparison is not, however, quite so limited. Many probabilities, which are incapable of numerical measurement, can be placed ... between numerical limits. And by taking particular non-numerical probabilities as standards a great number of comparisons or approximate measurements become possible. If we can place a probability in an order of magnitude with some standard probability, we can obtain its approximate measure by comparison” (Keynes, 1921:176; cited in Brady, 1983).

Keynes – in addition to this – presents supplementary corroboration for his own logical theory of probability:

“In fact underwriters themselves distinguish between risks which are properly insurable, either because their probability can be estimated between comparatively narrow numerical limits or because it is possible to make a “book” which covers all possibilities, and other risks which cannot be dealt with in this way and which cannot form the basis of a regular business of insurance – although an occasional gamble may be indulged in. I believe, therefore, that the practice of underwriters weakens rather than supports the contention that all probabilities can be measured and estimated numerically” (Keynes, 1921:24).

Arthmar–Brady (2016), assessing Keynes’s breakthrough based on his logical theory of probability, highlights that Keynes’s theory of probability is a logical, objective epistemic approach that is based on partial rather than on complete resolution. These results can be specified and operationalised in the form of any kind of logical premises. This can easily be applied to unique events, non-recurring events, irreversible events, singular events, infrequent events, frequent events, path dependency, sensitivity to initial conditions, emergencies, complex reasoning, attractive states, partial uncertainty or irreducible uncertainty, based on Keynes’s analysis of the weight of evidence.

Faced with the fact that after his TP of 1921, Keynes neither published any new work on logical probability nor took part in the continued development of the logical probability school of thought – initiated by him – we have to agree with Hársing’s (1971b:242) conclusion that Keynes regarded the creation of his preferred version of logical probability as being important as a means of underpinning the results of his specialist (economics) research.¹¹

¹¹ Although the *Treatise on Probability* was not published until 1921, it was essentially complete ten years earlier. This was the period in which Keynes developed his ideas on probability, and

Keynes pitted the complete future knowledge of classical economics against uncertainty. In what follows, we seek the answer to the question how close the relationship is, in Keynes's seminal works, between his own theory of probability and his theory of economics, and of whether continuity of thinking can be demonstrated between the *Treatise on Probability* (1921) and the *General Theory* (1936). To decide this, we must first take a closer look at the role of uncertainty in Keynes's conceptual framework.

4 THE ROLE OF UNCERTAINTY IN THE KEYNESIAN CONCEPTUAL FRAMEWORK

Uncertainty is a central category in Keynes's (1921) seminal work on probability, in which he describes this concept and phenomenon as multidimensional. Uncertainty features in this work with two independent definitions, with the two meanings deriving from the concepts of probability and weight. O'Donnell – invoking the model of logical probability – says that when the true value of a statement is unknown, we resort to its probability to indicate – with regard to the evidence provided – the appropriate degree of rational belief. This definition of uncertainty has a at its centre. But in an entirely difference sense this also lies in the centre of h , which relates to the degree of completeness of the relevant information on which the argument is based. We know that the data in our possession are not complete, and we are also uncertain as to the extent of this incompleteness. The uncertainty – in this sense – stems from the partial lack of relevant knowledge (O'Donnell, 1999a:259).

Rosser (2001) believed that Keynes's conception of uncertainty developed paradoxically over time. One reason for this was that Keynes presented several different arguments relating to uncertainty, encapsulating certain shifts in his views, increasingly emphasising that the chief characteristic of uncertainty is unquantifiable nature. Nevertheless, the starting point was his TP published in 1921, which served as the basis for his later views.

Keynes's article (1937) gives the most characterful explanation of uncertainty as he perceived it. Keynes started with “the price of copper and the rate of interest twenty years hence” (1937:214), and then went on:

“About these matters there is no scientific basis on which to form any calculable probability whatever. We simply do not know. Nevertheless, the neces-

therefore it significantly predates his work on the role of uncertainty in economics (Cf. HAM-
OUDA-SMITHIN, 1988).

sity for action and for decision compels us as practical men to do our best to overlook this awkward fact and to behave exactly as we should if we had behind us a good Benthamite calculation of a series of prospective advantages and disadvantages, each multiplied by its appropriate probability, waiting to be summed.”

Keynes deals with *four* versions of uncertainty, which means that he himself differentiated between the various degrees of uncertainty and did not consider fundamental uncertainty to be the only variant. The *first* group consists of events that have unknown outcomes, and an ex ante probability rate (or distribution). These are the sources of “probability knowledge”.

A paradigmatic example of this is gambling in a casino. For Keynes, the source of the probability rate is compatible with the frequency approach, as well as with the objective interpretation of probability. The *second* version – unlike the previous one – means uncertain events where there is no “scientific basis” whatsoever for the probability rate. These are events that are beyond scientific knowledge, regarding which only unsubstantiated estimates can be made. According to Knight (1921:225), this is always the case when dealing with decisions made under unique circumstances.

As the *third* group, Keynes concedes that there are events which lie between the two extremes; as an example, he puts forward events that have no fixed ex ante probability rate, but are subjected to a credibly informed scientific analysis with a variable degree of certainty. The *fourth* is a version that is applied for practical reasons when uncertain events are treated as cases of probable knowledge although, from a theoretical perspective, such an act cannot be proven. (cf. Backhouse–Bateman. 2006).

A comparison of the three seminal works gives an example of the changing substance of Keynesian uncertainty. In his GT of 1936, Keynes discusses “irreducible uncertainty”, in his correspondence with Townshend in 1938, “unrankable uncertainty”. the first concept, “low weight uncertainty” appeared in Keynes’s (1936) work. By “very uncertain” Keynes does not mean “very improbable” (Keynes, 1936:148 and Note 1). O’Donnell points out that, as a consequence of this, very uncertain corresponds to a very low weight; that is, situations in which there is a lack of completeness of the relevant information. The second, or “irreducible” meaning of radical uncertainty features in Keynes’s (1937) work. The key to this concept is Keynes’s doctrine of “unknown probability”. According to Keynes, the meaning of this is that we “simply do not know”; in other words, individuals have no knowledge of probabilities. The actors, owing to insufficient logical insight, are deprived of the ability to perceive the probability relationship. This is not due to any deficiency of intellect; in situations where *h* is exceptionally small, not even highly intelligent actors have the mental capacity for solving the logical relation-

ship between *a* and *h*. This context presents a good example of the power of human reasoning when scant data is available. In such cases, the uncertainty does not lend itself to being reduced to probability. The third is the “unrankable” version of uncertainty that comes from Keynes’s correspondence with *Townshend*¹², and which refers to the impossibility of generating a complete ranking (cardinal or ordinal) of alternative courses of action. This impossibility is related to the existence of incomparability between the probable values.

Given such a wide variety of definitions of uncertainty, *Koppl* (1991) justifiably concludes that it is difficult to make a credible judgement based on uncertainty, especially given fundamental (radical) uncertainty, which Keynes emphasised in his (1921) and (1937) works in keeping with the weight of the argument. When knowledge is “uncertain”, people are not capable of estimating probability, or at least not credibly; and they cannot demand more knowledge about the future. When knowledge is “certain”, it is not possible to obtain a good Benthamite calculation of future value, whether in the moral, hedonic or economic sense. If the uncertainty is sufficiently large, then we simply do not know (Keynes, 1937:213–214). When this version of uncertainty is present, the rational basis for action is substantially weakened. The “animal spirit” is needed to prevent economic actors from being stymied in their operation.

Rényi’s (1976) stance on the relationship between information and uncertainty gives an interesting illustration of the epistemic nature of uncertainty:

“As regards the concept of information, it is expedient (...) to introduce a related concept: the category of uncertainty. The result of an experiment whose outcome depends on the random is, to a greater or lesser degree, uncertain. Upon performing the experiment, this uncertainty ceases. The remaining uncertainty regarding the result of the experiment can be measured with the quantity of information that we obtain (on average) by conducting the experiment. The uncertainty, therefore, can be perceived as an information deficit (uncertainty is negative information), or conversely: we can interpret informa-

12 Townshend’s correspondence with Keynes started on 7 April 1937. Keynes provided Townshend with an extremely important clue to the mystery, which went unnoticed. *“But a main point to which I would call your attention is that, on my theory of probability, the probabilities themselves, quite apart from their weight or value, are not numerical. So that, even apart from this particular point of weight, the substitution of a numerical measure needs discussion”* (KEYNES, 1979:289, cited in BRADY, 2018). Keynes was explaining that the theory of probability that he applied in the GT in 1936, and which Townshend and Keynes were discussing in the letter of April 1937, was the theory of logical probability combined with the weighting of evidence. According to Keynes, the probabilities have to be non-numerical and indefinite by necessity in millions of cases; however, the non-numerical probabilities relate to the probability interval (BRADY, 2018).

tion as the elimination or reduction of uncertainty (information is negative uncertainty)”¹³

Most interpretations of uncertainty are epistemic, a good example of this being Davidson’s (1982) opinion that in reality there are many situations in which we are faced with “true” uncertainty regarding the future consequences of today’s choices. In such cases, the decision-makers see that neither today’s expenditure on the analysis of past data nor the present market indicators can be expected to offer reliable statistical or intuitive assistance in foretelling the future.

Recent decades have seen a growing recognition that uncertainty also has certain ontological aspects. If fundamental uncertainty is assumed, future states cannot be specified because these will be established now and in the future. This suggests that future states cannot be anticipated. Something that has happened in the past or is happening in the present will not necessarily also occur in the future. It is the irreversible and open nature of time and the malleability of the future, not the limited capabilities of the economic actors that lead individual actors to disregard the possible patterns of action or future states. *Dunn* (2000:428) stresses that individuals are the builders of the future. In an uncertain world, the future – prior to its formation – cannot be known, regardless of the calculation abilities attributed to individuals. It is not possible to know, *ex ante*, how any story will develop, and it matters not how much information and computing capacity a decision-maker has, the future can never be predicted *ex ante* with certainty (of probability).

In addition to the conditions of uncertainty, the expectations on which the decision rests are also dependent on the imagination and on intelligence, and on the narratives by which they are communicated; and they encapsulate feelings and emotions. According to *Bronk* (2009:221), imagination and creativity are not merely the main causes of ontological uncertainty, they are also important tools for describing uncertainty... The future has no precise vision, since this will be determined subsequently with innovations that have not yet been discovered and with decisions that have not yet been made, as well as the opportunities in this regard; market valuations only reflect our best views, the preferred narratives and the fleeting attitudes of optimism and pessimism (op. cit. 258).

13 A reduction in uncertainty can be interpreted as information, but a change in unexpectedness is not information; only the expected value of this quantity can be accepted as a quantity of information, and only because it is equivalent to a decrease in uncertainty (RÉNYI, 1976).

5 ANIMAL SPIRITS, EXPECTATIONS, INVESTMENT DECISIONS

5.1 The introduction of animal spirits

Animal spirits are a key category in Keynes's (1936) seminal work on economics. According to Kopp1 (1991), animal spirits come into the frame as a cause of action on the one hand, and as a subsequent source of instability on the other. Keynes believes that most of our actions cannot derive from "a mathematical expectation, whether moral or hedonistic or economic". Keynes felt that "most, probably, of our decisions to do something positive, the full consequences of which will be drawn out over many days to come, can only be taken as a result of animal spirits" (Keynes, 1936:161). He defined animal spirits as "a spontaneous urge to action rather than inaction" (1936:161)¹⁴.

Although Keynes saw the main thrust of the individual's behaviour as being to maintain a rational economic face, he was also aware of the limitations on the attainability of such. He saw the reasons for these as follows:

"Knowing that our own individual judgment is worthless, we endeavour to fall back on the judgment of the rest of the world which is perhaps better informed. That is, we endeavour to conform with the behaviour of the majority or the average. the psychology of a society of individuals each of whom is endeavouring to copy the others leads to what we may strictly term a conventional judgment" (Keynes, 1937:214).

According to Keynes, a lack of information and the general uncertainty of the future prevent entrepreneurs from forming scientific or rational expectations; but if they need to act, they substitute this with conventional expectations which then determine their investment decisions. However, precisely because this expectation is largely conventional, it is vulnerable to waves of optimism or pessimism, and the general state is the famous animal spirits (cf. Keynes, 1936:161–162). Keynes also warns that the actions inducted by the animal spirits are fundamentally *irrational*. He believed that rational action and probability are inseparable phenomena. In the relevant passage of his treatise, Keynes (1921:351) writes that "the probable is the hypothesis on which it is rational for us to act." People who are driven forward by animal spirits are not controlled by a more or less likely estimate; in this sense, their actions are irrational. Keynes took the view that rational actions must be based on rational belief. When people revert to the animal

¹⁴ Keynes's train of thought continues as follows: "Thus if the animal spirits are dimmed and the spontaneous optimism falters, leaving us to depend on nothing but a mathematical expectation, enterprise will fade and die – though fears of loss may have a basis no more reasonable than hopes of profit had before" (KEYNES, 1936: 162).

spirits, they are not acting on the basis of beliefs that are considered to be rational. Therefore, their actions are not rational.

Based on the foregoing, Koppl (1991) justifiably asks whether we need to take animal spirits seriously in economics. If we do, then is this not abandonment of an economic theory that is based on rationality? There is some evidence to suggest that “irrationalities” matter from time to time. The story of economic bubbles shows that investor behaviour is sometimes justifiably labelled as “irrational”, because it can and does influence market processes. Koppl emphasises that there is no proof that people are irrational by nature. Rather, the signs show that it may be useful to take the animal spirits seriously, seeking those economic conditions under which the impulsive side of human nature counts, and those conditions under which it does not.

Keynes asserts that the lack of information, and general uncertainty regarding the future, make it impossible for the decision-makers to form rational expectations, and this fact is pivotal with respect to their investment decisions. On this basis, Keynes does not conclude that every single actor forms his or her individual expectations that differ from those of all the other actors. Indeed, upon closer examination precisely the opposite is the case: the actors emulate each other, and thus they are members of a group whose members represent more or less the same viewpoint. This type of expectation, however, is based not on calculations, but on factors such as, for example, the state of the animal spirits. Rosser (2001) views the Keynesian perception of uncertainty as a fundamental and unquantifiable phenomenon to be the basis for why the “bird on the wing” of real capital investment is directed not by long-term rational expectations, which would not even be possible, but is driven by the essentially subjective and ultimately “irrational” animal spirits, a spontaneous urge to action in the face of uncertainty.

Hodgson (1985:13) confirms that irrational decision-making stems not from human nature, but from the circumstances surrounding the decision and action. He writes the following on this: “according to Keynes, human beings are rational but they live in a world where widespread uncertainty places severe limits on the capacities of individuals to make detailed, rational calculations about the future. These constraints derive not from the limited rationality of individuals but from the ubiquitousness of uncertainty”.

5.2 The role of expectations

Keynes makes a sharp distinction between short-term and long-term expectations. A short-term expectation “is concerned with the price which a manufacturer can expect to get for his ‘finished’ output given his general productive capa-

bilities.” It is thus very different from long-term expectation which is concerned with what the entrepreneur can hope to earn in the shape of future returns if he purchases ‘finished’ output as an addition to his capital equipment” (Butos-Koppl, 1995:46–47). From these definitions, Keynes concluded that a company’s daily output depends on its short-term expectations, whereas its investment in new capital is a function of long-term expectations.

Keynes’s theory of long-term expectations is based on his rationalism, and states that there is very little correspondence between expectations and the economic events. According to Butos–Koppl (1995), Keynes believed that economic expectations are subjective. However, the subjectivity of the expectations has more pronounced consequences in the case of long-term expectations than in the case of short-term expectations. While short-term expectations are closely associated with the realised values, long-term expectations are not formed by a rational calculation, because they do not “rest on an adequate or secure foundation”. (Keynes, 1937:218). All this leads us to conclude that, in his view, long-term expectations cannot establish a systematic relationship with fundamental economic reality.

Butos–Koppl (1995:59) perceptively concludes that, for Keynes, expectations regarding the future are states of belief. If these *belief states* reliably direct expectations, then they embody credible knowledge; but the reliable prediction of the future is not possible. “We cannot make any ‘calculated mathematical expectation(s)’ of future values, writes Keynes (1936:152), then goes on to say that “in a world where people plan for the future (...) most action is irrational action. ‘Most probably, of our decisions to do something positive,’ Keynes believed, “the full consequences of which will be drawn out over many days to come, can only be taken as a result of animal spirits, a spontaneous urge to action rather than inaction” (op. cit. 161). Keynes only gave actions the opportunity to “struggle with the dark forces of time and our ignorance of the future” (1936:157). Thus – according to Keynes – on modern asset markets, speculators’ long-term expectations create an atmosphere that generates nihilistic waves of pessimism and optimism that translate into waves of greater and lesser investment spending.

5.3 The investment decision

In his seminal economics work, Keynes (1936) dealt – in relation to long-term expectations (cf. Chapter 12) – with the knowledge of the future that could be necessary for making correct decisions and encouraging capital projects; he concluded that, because a certain knowledge of the future is unattainable, by their nature decisions relating to capital projects have to be based on a belief in the foundation of knowledge, which is flimsy at best.

Coddington (1982) believes that Keynes, in the context of the GT, presents uncertainty as an inherent part of investment decisions. This is the reason for Keynes's assertion that the foundation of knowledge for investments in the private sector is flimsy. The following passage (which we have already invoked more than once) sheds light on Keynes's concept of uncertainty:

“The sense in which I am using the term is that in which the prospect of a European war is uncertain, or the price of copper and the rate of interest twenty years hence, or the obsolescence of a new invention, or the position of private wealth-owners in the social system in 1970. About these matters there is no scientific basis on which to form any calculable probability whatever” (Keynes, 1937:214).

Investment decisions are based on beliefs regarding future circumstances which, however, have to be based on the conditions of the present and past. Accordingly, investment behaviour may show capricious fluctuations either as the present conditions change unpredictably, leading to irregular fluctuation with regard to anticipated future conditions, or through changes in the beliefs forming the basis for the decisions, without any corresponding changes in the actual conditions. Of these two scenarios, it is the second that leads to autonomous volatility in the aggregated expenditure arising from investment decisions.

In keeping with this, Coddington (1982:481) maintains that if changes in private investment are rooted in the spontaneous and capricious functioning of the human mind, then there is a solution to Keynes's problem: such a cataloguing would provide the reason why this type of expenses fluctuates autonomously instead of responding to changes in objective circumstances. This is the way in which subjectivist ideas show themselves in Keynes's GT.

It's worth pointing out that, from the perspective of the Keynesian argument, it is not really the fact of the uncertainty that is important, but rather how individuals are likely to react to the fact of the uncertainty. Accordingly, if the investment decisions are shrouded in great uncertainty, manufacturers respond to this for as long as possible by making the same investment decisions during this period as they did in the previous one (because the results of the previous decisions are what the decision-makers know something about). This does not result in greater stability than could be expected from complex calculations performed on a cognitive basis using privileged beliefs, or from forecasts with an indeterminate background. On this basis, the fact of uncertainty does not in itself lead to conclusions regarding the voluntary and unchecked behaviour of specified macroeconomic variables.

We have to agree with Weintraub's (1975) conclusion that Keynes made a breakthrough in economics with his GT, specifically by making the relationship be-

tween uncertainty and investment explicit; and the theoretical core of this relationship was already present in the TP.¹⁵ Another aspect of this theoretical innovation was that Keynes moved beyond games of chance and applied the language of probability to real decision-making situations. When evaluating alternative courses of action, individuals are driven by their views regarding the most probable outcome. The outcomes are manifest in the future; but they cannot be observed in the present. In this regard, Keynes considered it important to underline the following:

“The theory can be summed up by saying that, given the psychology of the public, the level of output and employment as a whole depends on the amount of investments [although a few other factors may influence output] ... it is they which are influenced by our views of the future about which we know so little (Keynes, 1937:221).

Keynes treated as fact the phenomena whereby 1) capital assets are long-lasting, 2) the desire to hoard money reflects the degree of our mistrust of the future, and 3) production needs time. These are all facts associated with a world in which time is important. In the course of our previous reasoning it became clear that time and uncertainty are intertwined; the former inevitably attracts the latter. Weintraub concludes that Keynes's system was dynamic in the traditional sense that it includes time as a material factor; thus, if investments are volatile due to uncertainty, there is not level of output or employment that can always be maintained. This is why Weintraub calls uncertainty an equilibrium phenomenon and can declare that Keynes was concerned with equilibrium problems (Weintraub, 1975:541).

Keynes believed that business calculations are deeply unreliable: “the outstanding fact is the extreme precariousness of the basis of knowledge on which our estimates of prospective yield have to be made” (Keynes, 1936:76). Keynes believed that the incompetence of long-term expectations did not cause difficulties in calmer times when corporate shares could not be “floated off on the Stock Exchange at an immediate profit”. (op. cit. 76). “Decisions to invest in private business of the old-fashioned type were, however, decisions largely irrevocable, not only for the community as a whole, but also for the individual” (op. cit. 76). The entrepreneur's attachment to his or her own capital might be seen as a burden on the investment when it's precise present value cannot be calculated. But business ventures are not launched “merely as a result of cold calculation.” (op. cit. 76).

¹⁵ According to JOAN ROBINSON (1973:3) “On the plane of theory, [Keynes's] revolution lay in the change from the principles of rational choice to the problems of decisions based on guesswork and convention.”

Thus, the irrational element has a positive effect on human actions. The animal spirits prompt people to act in a way that is socially beneficial, motivating the individual to invest.

In Keynes's opinion, the emergence of the stock exchange brought about a change, on which he wrote the following: "with the separation between ownership and management which prevails to-day and with the development of organised investment markets, a new factor of great importance has entered in, which sometimes facilitates investment but sometimes adds greatly to the instability of the system" (op. cit. 76). The new factor was speculation. The speculator does not try to measure present value, but the share price of the near future. Because the present value calculation is largely false, the speculator's estimates have no grounding in any assumed market reality on which they are based. Keynes argues that the professional trader wants to know the forthcoming changes in current asset prices and is not interested in long-term values.¹⁶

Investments associated with fundamental decisions become volatile, and concurrently with this the changes in expectations become substantial forces in the determination of economic activities.

For Keynes, the operationalisation of economic activity takes place by the calendar of historical time: When making their decisions, economic actors use the irreversibility of the past and the unpredictability of the future as references. In the words of Keynes himself, this can be expressed as follows: "... philosophically speaking, it cannot be uniquely correct, since our existing knowledge does not provide a sufficient basis for a calculated mathematical expectation. In point of fact, all sorts of considerations enter into the market valuation which are in no way relevant to the prospective yield" (Keynes, 1936:77). In line with Keynes's opinion, the long-term expectations on which our decisions are based do not only

16 BLACK'S (1986) paper introduces a modern version of speculative trading, which rhymes well with Keynes's description from his own era. "In my model of the way we observe the world, noise is what makes our observations imperfect. It keeps us from knowing the expected return on a stock or portfolio. (...) Noise makes financial markets possible, but also makes them imperfect. If there is no noise trading, there will be very little trading in individual assets. I do not believe it makes sense to create a model with information trading but no noise trading where traders have different beliefs and one trader's beliefs are as good as any other trader's beliefs. Differences in beliefs must derive ultimately from differences in information. A trader with a special piece of information will know that other traders have their own special pieces of information and will therefore not automatically rush out to trade. Noise trading provides the essential missing ingredient. Noise trading is trading on noise as if it were information. People who trade on noise are willing to trade even though from an objective point of view they would be better off not trading. Perhaps they think the noise they are trading on is information. Or perhaps they just like to trade" (op. cit. 529–531).

depend on our most likely forecast; they are also just as dependent on the confidence with which that forecast has been made.

Although long-term expectations remain constant for a long time, they are nevertheless exposed to sudden and violent changes that may at times be caused by (sometimes irrational) speculation, although they can also be triggered by psychological changes. Keynes – as we have shown above – presents his own theory in the form of “animal spirits”. He claims that these are the forces behind capital investments: “a spontaneous urge to action rather than inaction, and not ... the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities” (Keynes, *op. cit.* 81).

6 CONTINUITY OR DISCONTINUITY IN THE ROLE OF PROBABILITY IN THE KEYNESIAN CONCEPTUAL FRAMEWORK

The question of how Keynes’s GT relates to the fundamental premise of his main work on probability, the TP, has been open and disputes for many long decades. Gillies (1988) takes the view that, upon first consideration, the relationship could be very loose, because in the TP Keynes identified probability as the degree of rational belief. The degree of rational belief was a given for *a*, and for *b* it was something that can be calculated as a purely logical relationship, and this was the same for all rational individuals. This setup is very far removed from the influence of the animal spirits. Gillies nevertheless believes that there are passages of the TP that give a foretaste of what the GT has to say. Where Keynes discusses the measurement of probability the TP, he bases his reasoning on examples such as “quotes at Lloyd’s” (*op. cit.* 23), and he concludes that “no exercise of the practical judgment is possible, by which a numerical value can actually be given to the probability of every argument” (*op. cit.* 29).

Sharp differences of opinion emerged as to whether “there was continuity or whether there was a break in continuity” following Keynes’s 1921 TP. On one side there was the opinion that Keynes – chiefly in response to Ramsey’s criticism – switched from logical probability to a subjective probability approach (Bateman, 1987). The other side came to be dominated by a belief that Keynes did not adapt any alternative probability method that differed from logical probability, but continued his work within the original framework of the TP. This position is most assertively represented by O’Donnell (1990).

There is a marked difference between Keynes’s (1921) and Ramsey’s (1931) conception of probability, and this influences debates on the topic to this day. Ramsey was the first to describe the applicability of a subjective feeling as a means

of interpretation. He looked upon this approach as being complementary to the frequency interpretation of probability, which was an established theory at that time. Ramsey emphasised the measurability of the probability relationship. *He believed it was possible to arrive at probability values with a behavioural experiment; that is, he viewed reliance on the betting process as an aid for determining belief.* In Ramsey's view, belief can only be measured through a study of behaviour, and there is no sense in differentiating between the measurable and the non-measurable component (risk and uncertainty), because even if such a distinction is relevant, on a theoretical basis there is no adequate reason for avoiding the quantification of this segment, because the degree of belief is suitable for measurement. Ramsey anyway ruled out *a priori* knowledge of the probability of a claim: all he conceded was that the present feelings, paired with a knowledge of the observed evidence, can lead back to the initial feeling. Therefore, this initial feeling can be determined *a posteriori*. This was the ideological basis for Ramsey's (1926) severe criticism of the interpretation of logic articulated by Keynes. The following passage from Ramsey is often quoted to support this:¹⁷

“But let us now return to a more fundamental criticism of Mr Keynes's views, which is the obvious one that there really do not seem to be any such things as the probability relations he describes. He supposes that, at any rate in certain cases, they can be perceived; but speaking for myself I feel confident that this is not true. I do not perceive them, and if I am to be persuaded that they exist it must be by argument; moreover I shrewdly suspect that others do not perceive them either, because they are able to come to so very little agreement as to which of them relates to any two given propositions” (Ramsey, 1926:161).

The main thrust of Ramsey's criticism was that the version of the probability relation discussed by Keynes simply does not exist, and Ramsey's own procedure (betting) makes it much easier to find the “degrees of belief” held by people.

Keynes (1933), in an essay honouring Ramsey (after his death in 1930), wrote the following in response to Ramsey's criticism:

*“Ramsey argues, as against the view which I had put forward, that probability is concerned not with objective relations between propositions but (in some sense) with degrees of belief, and he succeeds in showing that the calculus of probabilities simply amounts to a set of rules for ensuring that the system of degrees of belief which we hold shall be a consistent system. Thus the calculus of probabilities belongs to formal logic. But the basis of our degrees of belief – or the *a priori* probabilities, as they used to be called – is part of our human*

¹⁷ RAMSEY first critiqued KEYNES's (1921) TP in his book review published in the 1922 issue of *Cambridge Magazine*, and later in his essay *Truth and Probability*, published in 1926.

outfit, perhaps given us by natural selection, analogous to our perceptions and our memories rather than to formal logic. So far I yield to Ramsey – I think he is right. But in attempting to distinguish ‘rational’ degrees of belief from belief he was not yet, I think, quite successful” (Keynes, 1933:300–301).

On this basis, it is safe to say that Keynes was prepared to accept Ramsey’s opinion on several points; but it was clear that the two of them did not agree on everything. Bateman (1987) commented, on Keynes’s views regarding probability, that Keynes had adopted a subjective interpretation of probability. After the above Keynes quote, Bateman, wrote the following:

“While Keynes (1921) had originally advocated an objective epistemic theory or probability, he was not willing to accept a subjective epistemic theory...” (Bateman, 1987:107).

What is less debatable with regard to Bateman’s opinion is that Keynes moved away from the logical interpretation of probability; however, the direction and extent of that shift demands more detailed argumentation, especially in view of Keynes’s theoretical position thus established. We should be clear that all Keynes wrote in his reply to Ramsey was that Ramsey was right to say that the “degree of belief” is essentially rooted in human nature and not in form logic.

As regards the question of continuity or discontinuity, we should regard O’Donnell’s view on this as definitive. He made two assertions: firstly, *Keynes’s thinking continued to be based on the framework assumptions of the TP*; secondly, *there was an internal shift within the constraints of these assumptions after 1931*, whereby the importance of the indeterminate domain and the weight of weak rationality increased, while the significance of the determinate domain and the strong rationality decreased.¹⁸

Gillies (1988) also poses the question of whether Keynes’s view on probability changed over the years. He concludes that Ramsey’s criticism of Keynes’s views moved Keynes into an intermediate position between his original logical interpretation of probability and Ramsey’s subjective probability theory. Gillies defines Keynes’s new theoretical position as constructing a so-called *intersubjective probability theory*, making use of Keynes views on the long-term expectations of entrepreneurs. Due to the lack of information and to general uncertainty, companies have a tendency to copy each other, following the crowd, and thus amplifying the “animal spirits” which Keynes describes as often being the cause of sudden

¹⁸ O’DONNELL (1990b), albeit without providing an explanation, nevertheless claimed that there is strong evidence for the shift of emphasis in Keynesian thinking that was already running its course – like an underground – in the mid-1920s. He believes that Ramsey’s effect on Keynes was to reinforce this trend.

changes in economic activity. Based on the foregoing, we can state that Keynes was closer to the intersubjective epistemic theory than the subjective epistemic theory championed by Ramsey. Lawson (1985) rightfully concludes that intersubjective probability was closer to Keynes's earlier thinking, and a group of intersubjective probability occupies an intermediate position between rational belief (early Keynesian thinking) and subjective belief (Ramsey).

An important point of Ramsey's (1926) criticism was the dismissal of the principle of indifference; with regard to this he notes the following:

“To be able to turn the Principle of Indifference out of formal logic is a great advantage; for it is fairly clearly impossible to lay down purely logical conditions for its validity, as is attempted by Mr Keynes” (Ramsey, 1926:189).

This opinion may have had a role in the fact that Keynes, who based the TP's whole train of thought on the principle of indifference, completely repudiated this principle in the GT. Keynes wrote the following on this in his seminal work of economics:

“Nor can we rationalise our behaviour by arguing that to a man in a state of ignorance errors in either direction are equally probable, so that there remains a mean actuarial expectation based on equi-probabilities. For it can easily be shown that the assumption of arithmetically equal probabilities based on a state of ignorance leads to absurdities.” (Keynes, 1936:152).

Gillies (1998) also points out that Keynes still did not capitulate to Ramsey, and he had doubts as to whether Ramsey provided a satisfactory explanation for the differentiation between the degree of belief and the degrees of rational belief. Keynes's original probabilities; that is, the degrees of rational belief, were the same in respect of all existing individuals. Ramsey's probabilities or degrees of subjective belief were associated with a given individual, and thus they changed from one individual to the next. The expectations of entrepreneurs in a given economy, in keeping with Keynes's later theory, occupied a position somewhere between the two. In his review of the TP, Ramsey (1922) asserted that probability must be precise, exact and mathematical. According to Brady (2018), Ramsey's criticism prompted Keynes to emphasise even more strongly the individual's own opinion as the basis for the probability calculation, and he was less emphatic that this belief was rational. However, Keynes's theory did not stand or fall on the opinion regarding the degree of our belief as a logical relation. The core of his theory – in terms of when and how we are capable of measuring and comparing the various probabilities – did not change. Unlike Ramsey, he was by no means certain that probabilities are always one-dimensional, measurable, quantifiable or comparable entities.

Since Ramsey believed firmly that every probability is a numerical value, he regarded Keynes's probability theory, both as a whole and in its details, as being a purely qualitative interpretation; and thus he reasons that in the absence of numbers Keynes's analysis is merely a qualitative and comparative analysis that can only be applied with limitations. The question of whether this perceived relationship between Keynes's early work (TP) and his later uncertainty conception represented continuity or discontinuity is discussed in-Depth by Lawson (1985:914). There is, however, a thread of the problem that receives very little attention. Hamouda-Smithin (1988) attributes particular important to the position taken by Keynes, emphasising the fundamental difference in treatment of the topic under study between the natural sciences and the so-called "moral" sciences (social science), and thus its suitability, which Keynes described in connection with his "atomic" and "organic" hypotheses. A study of this aspect helps us understand the role of uncertainty in social relations, and also contributes to acceptance of Lawson's theory of "social interactionism" (Lawson, 1985:926).

Here, moving beyond the dichotomy of continuity versus discontinuity, we will examine the evolutionary process in which items that were present in the TP, but later evolved in Keynes's subsequent works, were adapted specifically for economics; namely, the applicability of the atomic hypothesis in the moral; that is, the social sciences.

According to the TP, the most important types of arguments used for establishing probability relations are induction and analogy. The basis for Keynes's mode of reasoning is the atomic theory, which he describes as follows (Keynes, 1921:287):

"The system of the material universe must consist, is this kind of assumption is warranted, of bodies which we may term ... legal atoms, such that each of them exercises its own separate, independent and invariable effect, a change of the total state being compounded of a number of separate changes each of which is solely due to a separate portion of the preceding state. ... Each atom can ... be treated as a separate cause and does not enter into difference organic combinations in each of which it is regulated by different laws."

On the other hand, it is conceivable that the atomic theory is not confirmed, and Keynes describes this case as follows:

"Yet there might well be quite different laws for wholes of different degrees of complexity, and laws of connections between complexes which could not be stated in terms of laws connecting individual parts. In this case natural law would be organic and not, as it is generally supposed, atomic" (op. cit. 287).

Hamouda-Smithin points out that the above quotes contain no reference to economic or social science. In other parts of the TP, Keynes expresses the opinion that a clear distinction must be made between the natural sciences and the moral

or social sciences; and the atomic hypothesis may have a role in the former at any time, it is categorically inappropriate in the latter. On this, Keynes wrote the following in the TP:

“The atomic hypothesis which has worked so splendidly in Physics breaks down in Psychics. We are faced at every turn with the problems of Organic Unity, of Discreteness, of Discontinuity – the whole is not equal to the sum of the party, comparisons of quantity fail us, small changes produce large effects, the assumptions of a uniform and homogeneous continuum are not satisfied” (op. cit. 262).

Towards the end of the 1930s, Keynes returned to the atomic-organic dilemma that resulted from the process of change that occurred in Keynes’s conception of uncertainty between the mid-1920s and the end of the 1930s. During this period, Keynes’s view on uncertainty were “radicalised”; the role of indeterminateness and the fundamental grew. A letter dated August 1938 refers to this:

“If we are dealing with the action of numerically measurable independent forces, adequately analysed so that we were dealing with independent atomic factors, ... we might be able to use the method of multiple correlation with some confidence. ... In fact we know that every one of these conditions is far from being satisfied by the economic material under investigation” (cited in Hamouda–Smithin, 1988).

Based on the above quotations, Keynes clearly puts forward the view that the atomic hypothesis does not apply the world of social relations. The elements of this world do not function as “legal” atoms, striving to exert their own independent effect under all circumstances, but yield willingly to various laws in all the alternative configurations of the system. On this basis, Keynes’s view on uncertainty can only be understood in relation to this vision of the social process.

7 COMPETING INTERPRETATIONS OF PROBABILITY IN TWENTIETH CENTURY

7.1 The redefined idea of relative frequency

Classical probability theory– from the late 19th century – came in for criticism due to the non-fulfilment of the principle of indifference and the principle of additivity, and the narrow scope for application of the theory. The most forceful challenge to the classical interpretation of probability came from Keynes’s seminal work on probability, laying the foundations for the system of logical probability. The great thinkers’ differing views on probability point to a multiplication of probability conceptions. This is ultimately why competing concepts emerged as challengers

in the twentieth century. Most economics thinkers were relatively unaffected by this competition, and scholars of this discipline maintained their imprecisely defined “objective” and “subjective” analyses.

Paradoxically, the challenging view that had the greatest impact was the “relative frequency” interpretation of probability put forward by Richard von Mises (1928) and *Reichenbach* (1961). The decline of the classical probability interpretation, and the emergence and spread of the science of statistics and mathematical statistics, led to a new interpretation of probability which – building on a solid mathematical foundation – amounted to a redefinition of the doctrines of the classical probability theory. In this theory, probabilities are associated not with individual results but with event types, and the theory itself takes an objective approach. The essence of the new approach can be expressed almost exactly in the same way as that of the classical probability interpretation: Under the “relative frequency” approach, the probability of a given event is the relative frequency of its occurrence in any trial in an infinite chain of similar trials.

The basis of Richard von Mises’s probability theory is the concept of the *collective*. The rational conception of probability, in contrast to probability as used in everyday conversation, only receives a precise meaning if the collective to which it is applied is precisely defined in every case. This is when probability has a real meaning with respect to a given collective. The collective essentially consists of a series of observations that continue for an indefinite period. Every observation ends with the recording of a certain property. The relative frequency with which a specified property occurs has a limiting value in the series of observations.

According to Hauwe (2011), Richard von Mises regarded the frequency approach to probability theory as a science of the same order as geometry or theoretical mechanics, because he believed that probability should be based on facts and not a lack of them. The frequency theory links probability with the real world through the observed objective facts (or data), with special regard to the recurring facts.

In the logical approach discussed in more detail above, the probability theory emerges as a part of logic, as the extension of deductive logic to inductive cases. In contrast to this view, a proponent of the frequency approach sees probability theory as a mathematical science, like mechanics, but with a different band of observable phenomena. Hauwe emphasises that this means probability cannot be interpreted in this way in an epistemic sense. It is not the absence of knowledge (uncertainty) that lays the theoretical foundation for probability, but the observation of a high number of events.

According to Hársing (1965), Richard von Mises sees relative frequency (statistical probability) as the exclusive form of probability. He defines probability as the limiting value of relative frequency obtained through the infinite repetition of a trial.

He excludes the problematics of moral decisions from the field of probability. In his opinion, the concept of probability is only applicable in the following three areas: games of chance, insurance transactions and mechanical and physical phenomena. The most important circumstance is that Richard von Mises rejects the concept of logical probability on the basis that it is subjective in nature (Richard von Mises, 1928:10–11).

In his critique, Hauwe (2011) also mentions that probability in economics is not a manifestation of physical entities as Richard von Mises supposes when constructing his theory. The empirical underpinnings of probability are missing in the economic sense, for example with respect to objective frequency probability. Richard von Mises is naturally aware of the fact that the frequency concept is not applicable in the moral sciences, because in the absence of events the conditions would be fulfillable as a collective. He wrote the following on this:

“Extending the validity of the exact sciences was a characteristic feature of excessive rationalism in the 18th century. We do not intend to make the same mistake” (Richard von Mises, 1928:76).

The main flaw in this theory is that it is too narrow, as probability is used in many important situations; but among these there are none in which the empirical collective can be defined in an economics context. The definition is too narrow for application in economics.

Hauwe (2011) identifies mismatches between the theories of Richard von Mises and Keynes. For Richard von Mises, probability is a part of empirical science; for Keynes, on the other hand, it is an extension of deductive logic. Richard von Mises defines probability as frequency with limiting values, and Keynes as a degree of rational belief. For Richard von Mises the probability axioms are derivable from two empirical laws by abstraction, while for Keynes they can be obtained through direct logical intuition. Richard von Mises believes we can only evaluate probabilities that are within empirical collectives, and only these probabilities have scientific value. *For Keynes, all probability obeys the same formal rules and plays the same role in our thinking.* Certain special aspects of the situation permit us to assign numerical values in some case, but not generally. By virtue of his recognition that probability frequency does not cover everything that we think about probability, Keynes’s position is close to the view of Ludwig von Mises.

While the frequency theory of probability relates to the cardinally measurable degree of probability, case probability – according to Ludwig von Mises (1969) – does not lend itself to any form of numerical assessment. In keeping with this view, case probability focuses on the individual events, which are not as a rule parts of a series, and case probability is only measurable in the ordinal sense; case probability has no cardinal value.

Both Ludwig von Mises and Keynes accepted the epistemic interpretation of probability, but Richard von Mises unambiguously recognises the objective theory of probability. Ludwig von Mises's and Keynes's views amount to an argument that the economic interpretation of probability suggest that it is more epistemic than objective by nature. At the same time, both Ludwig von Mises and Keynes, in their own ways, recognise the existence of unmeasurable (or non-numerical) probabilities, as well as the epistemic and scientific legitimacy of these, while the customary measurable probabilities have a defined numerical value in the $[0, 1]$ interval. Although Richard von Mises conceded that there was a generally accepted concept of probability that was not covered by his theory of frequency, he nevertheless insisted that there is only one conception of probability that has scientific relevance. To express his views in other words: *There is only one scientific approach to the subject, and there is no room for the purely qualitative conception of probability. Although the applicability of the frequency theory of probability is called into question in several areas of natural science, there appears to be agreement in favour of two conclusions: according to one, in any case the scope of frequency theory is not broad enough for economics; according to the other, the fact that in economics probability is a qualitative, and not a numerical concept, is both necessary and scientifically legitimate.*

Hársing's (1965) evaluation of Richard von Mises's theory confirms our supposition that this theory – in essence – is a redefinition of the 19th-century frequentist conception of probability. According to Richard von Mises, probability calculus is the theory of recurring cases of certain pseudo-random or random events or series of events, like the rolling of dice. These series are defined by two axiomatic conditions as a “pseudorandom” or random series: One of these is the convergence axiom (or boundary axiom), and the other is the axiom of randomness. If a series of events fulfils both conditions, then to use Richard von Mises's terminology it makes up a “collective”. A collective – put simply – is a series of instances or events that could theoretically continue indefinitely.

The convergence axiom assumes that with the lengthening of the series of events the frequency series approaches a defined threshold value. Richard von Mises uses this axiom because for the purpose of application we need to make a frequency value certain. For Richard von Mises, probability is another word for “relative frequency value in a collective.” In his approach the concept of probability is applicable to a series of events; this restriction is diametrically opposed to the Keynesian position, and therefore it is entirely unacceptable if that is taken as the starting point.

The two axioms used by Richard von Mises to define the “collective” have come in for strong criticism, which Hársing believes is not entirely unjustified. The linking of the convergence axiom and the randomness axiom, in particular, were

criticised on the basis that it is not permissible to apply the concept of the mathematical threshold value or convergence to a series which – by definition (viz. due to the randomness axiom) – cannot be subordinated to any kind of rule.¹⁹

Reichenbach (1961) only recognises statistical probability, which – in his conception – is the limiting value of the relative frequency of random events. Accordingly, he believes that one-off events have no probability. Despite the fact that he ultimately only recognises statistical probability, Reichenbach also discusses logical probability. He believes, however, that logical probability is secondary in nature and can be traced back to statistical probability. It only differs from the latter in the fact that it is not based on the relative truth frequency of the events themselves, but of the statements made about them.²⁰

Reichenbach regards the statistical approach of Richard von Mises to be the only possible interpretation. Thus, ultimately, Reichenbach's logical probability is nothing other than the logical interpretation of Richard von Mises's approach. As we have just noted, the probability that a statement is true – in Reichenbach's conception – represents the frequency with which that statement is true (Reichenbach, 1961:319-326). If, like Reichenbach, we perceive logical probability as a generalisation of classical logic, then the greatest difficulty is caused by the linking of the concepts of truth and logical probability. It is known that truth (in an epistemic sense) is the relationship between the facts and a statement. Logical probability, on the other hand – in Reichenbach's conception – describes the relations of statement-sequences.

Reichenbach initially viewed probability as a limiting value in the mathematical sense, similarly to Richard von Mises. Later, when it was demonstrated that this approach leads to difficulties, he modified his conception so that the threshold value featured in the concept was not strictly mathematical in nature, but a so-called practical limiting value, the existence of which is based on the laws governing reality and the determinateness of the phenomena.

19 An a_n series can have a limit value if number A is such that an infinite number of items in the series fall into its arbitrary domain, and only a finite number fall outside of it. In this case number A will be the limit value.

20 According to HÁRSING (1971a), this approach does not differ significantly from the widely held view that probability is the centre of fluctuation of relative frequency and goes beyond Reichenbach's often unilaterally empiric attitude.

7.2 The evolution of logical probability and its variants outside economic thought

Following publication of the TP, Keynes did not concern himself at all with the theoretical problems of probability; nevertheless, the issue of probability did reappear – albeit indirectly – in the GT, in connection with the formation of expectations. It should also be noted that Keynes did not participate in the scholarly efforts that came after the breakthrough in logical probability that he initiated.

According to Carnap (1950) there is not one, but two concepts of probability; one is empirical, the other logical in nature, and both are objective. He considered the logical interpretation of probability to be one of several possible definitions rather than the only permissible one. In Carnap's view, one of the probability concepts is "probability 1", which describes the relationship between statements (specifically the degree by which a statement logically strengthens other statements); the other concept is "probability 2", which relates to the relationship between classes of events. Probability 1, therefore, is one example of how probability is viewed as a relational property of statements.

Amsterdamski (1965) highlights that, for Carnap, probability is the degree of confirmation of statements, while for *Jeffrey* (1939; 1954) it is the degree of rational belief. According to Amsterdamski, Carnap considers "degrees of rational belief" to be a bad term, because in Carnap's view the substance of Keynes's and Jeffreys's theory is that probability is nothing other than the degree by which a statement reinforces other statements; in other words, we are talking about logical probability. Neither Jeffreys nor Keynes recognise the existence of the two probability concepts, so as far as they are concerned the probability statements were never about what kind of events are probable; they only ever talk about which statements receive the most robust confirmation from the information available to the subject. Carnap identifies the concept of rational belief – from Keynes's and Jeffreys's theory – with the principle of his "degree of confirmation" and identifies the whole conception with logical interpretation.

Logical probability – in Carnap's approach – is created from rational belief by abstracting from the holder of the rational belief, as an epistemic subject, and only taking into consideration the logical relationship between certain past results of observation (the evidence) and the new knowledge (hypothesis). Thus, logical probability is simply the degree of confirmation (Hársing, 1971). Carnap refers to the version of probability devised by him as inductive logic.²¹

²¹ The classic version of intuitive deduction is generalisation. To use the terminology of mathematical statistics, generalisation is simply the drawing of conclusions, from a statistical sample, regarding the population (group) as a whole.

Carnap defines the degree of confirmation in a semantic system. In this, a numerical value is assignable to every statement. If we know, for example, the numerical value assigned to statements k and a , the $P(a/k)$ value can be defined; that is, the extent to which k confirms a (Carnap–Stegmüller, 1954:138–198).

Pólya (1949) applies the premises of abstract probability calculus in a comparative sense, for investigation of so-called plausible conclusions. Pólya – like Carnap – recognises the legitimacy of the two interpretations of probability, but just like Keynes he regards logical probability as the degree of rational belief. He, however, categorically rejects the quantitative interpretability of logical probability. On this basis, Pólya recognises two qualitatively different variants of probability: statistical probability and plausibility (plausibility being the degree of confirmation of hypotheses). “Viewed from the first perspective” he writes, “the theory of probability is the theory of observable phenomena of a certain type, the theory of random mass phenomena, and probability itself corresponds to relative frequency theory within a long series. From the second perspective, probability theory is the logic of plausible conclusions and probability is the degree of rational belief”. Hársing (1965) points out that the first position is close in essence to the views of Richard von Mises and the second is closer to those of Keynes (Pólya, 1949:28; Hársing, 1965:953).

The “propensity” concept of objective probability states that probability is a propensity or tendency of nature that typifies a certain event once without necessarily being related to longer-term frequency. It is important to note that these propensities are presumed to be objectively extant, if only in the metaphysical world. In this approach, probability really is the extent of absence of knowledge of the conditions that could influence the toss of a coin, and thus it merely reflects the sentiment relating to the trial.

Gillies (1988) takes propensity theory to be any approach that assumes objective probability but is not a relative frequency-based interpretation. Popper’s (1934) aim in creating propensity theory was to also be capable of assigning an objective probability to singular events²². Gillies criticises this view that objective probability can also be attributed to singular events; if we can find a narrowest reference class through which the probability of the singular event can be determined as the relative frequency of similar types of event in the given class. The probability of occurrence of a singular event depends on how we describe then event, and thus also on what background information we have regarding a particular

22 HÁRSING (1971a) believes that the physical motivation for this effort was quantum mechanics. POPPER (1997) – in his early theory – takes a set of initial generating conditions and regards them as having a certain tendency – a propensity – to “generate” the observed frequency.

event. According to Keynes (1921), any additional knowledge we may possess has an even greater role than our statistical knowledge: in certain cases, the latter may be completely modified by the former in relation to a singular event.

8 THE IMPACT OF PROBABILITY THEORY ON KEYNES'S ECONOMICS THINKING

In itself, the fact that in the analysis of long-term expectations set out in the GT we were able to refer, on several strands of enquiry, to the Keynes's probability concepts and arguments found in his TP, shows that these probability labels could have been the roots of his ideas relating to expectations. The premise that Keynes's thoughts on probability served as the basis for his theory of economic expectations is an accepted proposition among post-Keynesian thinkers²³. According to O'Donnell (1990/b), Keynes's (1921) TP is the appropriate starting point for understanding how the GT addresses uncertainty, expectations and behaviour. O'Donnell believes that we cannot find a precise parallel; what we do find, however, is an intermediated parallelism: one factor is the transition from the philosophical to the economic plane, while the other is the shift towards non-determinedness within the constraints of Keynesian philosophy.

In stark contrast to the dominant role of expectations, probabilities have a subordinate role in the GT. Known probabilities – whether numerical or non-numerical – are not fundamental concepts in the GT; the probability categories are not of central importance in this work on economics. In Keynes's GT, expectations are a general behavioural concept, and not probabilities. O'Donnell highlights that the actors always have expectations, but they do not always have probabilities. The expectations may have the nature of probabilities, they may be objective or subjective, strongly rational or weakly rational, or even irrational. Keynes's GT is primarily supported with induction, and this inductive approach has two parts: The first is the extrapolation of knowledge into the future; the second is the modification of this extrapolation in the light of specific, anticipated changes. O'Donnell believes that the second element is the truly important one, because *Keynes recognised that the extrapolation demanded by rational behaviour has to be altered if there are grounds to believe that the future will differ from the past.*

O'Donnell (1990b) also emphasised that Keynes (1921), in his work on probability, regarded it as a fundamental principle that philosophy and methodology perform

²³ Examples include the works of O'DONNELL (1989: 247-272) and CARABELLI (1988). SHACKLE'S (1972) *Epistemics and Economics* also deals with this topic.

a controlling function in economic argument. Keynes viewed reality as a primarily qualitative entity in the sense that the objective qualities of life do not have “numerical” quantifiability or a formalised character. The preliminary qualitative logical analysis must precede the quantitative investigation. Because both the relative and the absolute nature of probability suggests that they do not necessarily exist as a part of material reality, there is a need for *a priori* consideration. Lawson (1988) comments on Keynes’s thinking as follows:

“...throughout his total contributions [Keynes] is explicit that such a *priori* thought is considered always to be open to constant modification and correction through continual interaction with experiences of the real world” (Lawson, 1988:56).

Of particular importance here is Keynes’s comment to the effect that *probabilities must be made contingent on current uncertainties and knowledge, without regarding probabilities as relative frequency, since this will change after it has emerged*. Moreover, Keynes’s focus on the fundamentally qualitative nature of reality suggests that both informal argument and intuitive judgement are necessary for economic reasoning.

The concept of rational belief has a key role in Keynes’s works on probability and economics. He saw *two paths* to the attainment of rational belief regarded future prospects if perfect knowledge was not available. The *first* is based on the formation of probability, which can be arrived at either through uncertain information or a “doubtful argument” (Keynes, 1921:3). In the *second* case, it is impossible to define rational belief. In this event, actions are determined by the animal spirits. These are precisely the two types of uncertainty that classical theory rules out with the assumption that individuals have full or certain knowledge of what Keynes calls the “primary proposition” that a person sets out to validate.

Skidelsky (2011) offers a convincing argument as to why uncertainty was the main motif in Keynes’s work. Skidelsky believes that principal reason was that *the future cannot be forecast because it is open*. On this, he writes the following: “It is ‘open’, in large part, because it depends on our intentions and beliefs, and on the organic nature of human life. In talking about irreducible uncertainty Keynes does not just have in mind ignorance of the relevant probabilities, but genuine ontological indeterminacy: some probabilities are not just unknown, but non-existent” (Skidelsky, op. cit. 3). Keynes essentially believed that this is only relevant in areas that are characterised by risk rather than uncertainty, and therefore the investment markets are ruled out.

Brady (1983) recognises the most consistently that Keynes did not oppose attempts at approaching probabilities with estimates between lower and higher

thresholds or limits (op. cit. 27). This argument is related to the following extract from Keynes:

“It is evident that the cases in which exact numerical measurement is possible are a very limited class (...) The sphere of inexact numerical comparison is not, however, quite so limited. Many probabilities, which are incapable of numerical measurement, can be placed nevertheless between numerical limits. And by taking particular non-numerical probabilities as standards a great number of comparisons or approximate measurements become possible. If we can place a probability in an order of magnitude with some standard probability, we can obtain its approximate measure by comparison” (Keynes, 1921:160; cited in Brady, 1983).

While Keynes recognises that actual, precise numerical measurement is limited to identical probabilities through application of the principle of indifference (Keynes, 1921:Chapter 4); however, in the next chapter (5) he concludes that “Many probabilities, which are incapable of numerical measurement can be placed between (higher or lower) numerical limits, comparing them with various non-numerical (or numerical) probabilities selected as the standard”.

Keynes believed that logical probability, as a qualitative category, is the most suitable for describing the chances of occurrence both of series of events and of singular events, and that neither classical probability nor the frequency variant of probability is appropriate for this. Here is Keynes’s (1921) argument supporting this position:

“In fact underwriters themselves distinguish between risks which are properly insurable, either because their probability can be estimated between comparatively narrow numerical limits or because it is possible to make a “book” which covers all possibilities, and other risks which cannot be dealt with in this way and which cannot form the basis of a regular business of insurance – although an occasional gamble may be indulged in. I believe, therefore, that the practice of underwriters weakens rather than supports the contention that all probabilities can be measured and estimated numerically” (op. cit. 24).

In many respects, Keynes referred to himself an unconditional adherent to estimating the probability interval, and thus he rejected validation of the additivity criterion under all circumstances. *Keynes’s argument was that probabilities are primarily intervals and not singular numerical values or ordinal rankings.* This is confirmed by the following passage from Keynes:

“If we pass from the opinions of theorists to the experience of practical men, it might perhaps be held that a presumption in favour of the numerical valuation of all probabilities can be based on the practice of underwriters and the willingness of Lloyd’s to insure against practically any risk. Underwriters are

actually willing, it might be urged, to name a numerical measure in every case, and to back their opinion with money. But this practice shows no more than that many probabilities are greater or less than some numerical measure, not that they themselves are numerically definite. It is sufficient for the underwriter if the premium he names exceeds the probable risk. But, apart from this, I doubt whether in extreme cases the process of thought, through which he goes before naming a premium, is wholly rational and determinate; or that two equally intelligent brokers acting on the same evidence would always arrive at the same result” (Keynes, 1921:22–23).

When Keynes refers to knowledge and being informed, he means these true statements, regardless of whether they are direct or indirect knowledge. It is also clear that the rational belief encompassed by his theory is ultimately based on knowledge and thus on the truth. As Ramsey (1931:190) puts it, Keynes’s method was based on the fact that it only certified probable belief in relation to certain knowledge.

Right up until the middle of the 20th century, the economics thinkers concerned with uncertainty, risk and probability deliberately embraced complexity, and used probability to represent it. The peak of this thought process was Keynes’s TP of 1921, GT of 1936 and GTE of 1937; in these, probability and uncertainty appear as qualitative properties of decision-making, a mode of thinking suitable for covering economics as completely as possible. Following this – especially with the redefinition of the frequency theory of probability – the principles of measurability and quantifiability became dominant, and complexity was expressed with probability distributions, expected values and standard deviation as *compressed values*. Through this, the range of analytical possibilities was expanded but the complexity disappeared from the approaches. This process can also be described as the avoidance of complexity. *An important question is what led to the simultaneous acceptance and avoidance of uncertainty in the mainstream of economics.*

The rise of formalisation in economics coincided with the decline of the uncertainty conception, and the main reason for this is clear: It is difficult to incorporate uncertainty, as a non-quantitative phenomenon, into the formalised models. Therefore, numerous representatives of the mainstream simply purged this concept from their theories. Lucas (1977:15) wrote that “in cases of uncertainty, economic reasoning will be of no value”. This puts us in mind of Arrow’s (1951:417) analysis of Knight’s concept of uncertainty, which reaches the conclusion that “measurable probability cannot be established for such cases”. In this context, Lucas and Arrow confirm that the economic reasoning and the theory *must be quantitative*. In this regard, they have disregarded Knight’s and Keynes’s objection that uncertainty is not quantifiable. The waning of Knight’s and Keynes’s concept of uncertainty is attributable to a multitude of factors in the economic

mainstream, including the fact that models are expected to yield forecasts²⁴. *In economics – with the emphasis on testing and forecasting – he was only able to make uncertainty manageable, in order to reduce risk, by giving it a calculable form.*

With the emergence and rise of mathematical formalisation within economics, thinkers exploring expectations and uncertainty chose abstraction over complexity. Rowley–Hamouda (1987) believe that this shift was strengthened by the hope that the formulation, in itself, could be successful in making analytical solutions assignable to mathematically formed phenomena.

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24 The pervasiveness of formalisation naturally also gave rise to doubtful and critical opinions. KESLER (2007) comments that “Today, the identity of economics as a discipline is built on a particular method and mode of reasoning, not on the subject under consideration...” (op. cit. 118).

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