PIT AND TTC PROBLEMS RELATED TO IRB PD PARAMETER ESTIMATION IN THE LIGHT OF SUPERVISORY REVIEWS

Gábor Nagy – Gergely Biró

ABSTRACT

Financial institutions have to cover their risks by solvency capital. In general, among credit institutions, the most important risk is considered to be credit risk. Since Basel II, the regulator has also allowed the calculation of risk-sensitive capital requirements by applying the IRB approach (Internal Rating Based Approach). The method itself is widely known. In this publication, we deal with PD (probability of default) out of the parameters of the IRB approach. We focus on the cases in which the PD calculation methodology used by credit institutions fulfils the capital requirements at the appropriate level expected by the regulator. Of course, the problem does not only exist in the first pillar through the application of IRB, but it is also relevant in the case of credit risk portfolio models in the second pillar.

JEL codes: C4, G2

Keywords: TTC PD, PiT, PD, Capital Requirement, Cyclicality, Stability, Macroeconomic Factors, Modelling

1. MOTIVATION

In connection with supervisory reviews, it was problematic on many occasions that the PDs often used by institutions were too low during economic boom, as they typically had PiT (Point-in-Time) character. As a result, credit risk capital requirements have considerably decreased, as well. By contrast, in the event of economic downturn, the opposite happens. The level of PiT PDs is increasing, which leads to higher capital requirement level. In relation to these reviews, supervisory authorities are often concerned that in the case of economic boom, the IRB capital requirement calculated at 99.9% confidence level does not cover unexpected losses.

Vasicek's model (2002), on which the IRB capital function is based, as well, uses unconditional PD to calculate the conditional PD (i.e. the PD in a certain state of the economy, e.g. under stress). Although the Vasicek model defines unconditional PD (in the mathematical model and the framework system), the meaning of unconditional PD is less obvious in practice. It is important to emphasise that the Vasicek model is basically a simplified model (it determines the distribution of loss assuming that the maturity, probability of default and asset correlation of the portfolio elements are the same). At the same time, the currently effective IRB approach used for the calculation of capital requirement is based on the Vasicek model.

Consequently, despite criticism, it is important that ultimately, institutions should calculate prudent capital requirements by using this model. In this publication, we present methods by which the PD that serves as input for the IRB approach can be calculated in a way that results in a prudent, stable and sufficiently conservative capital requirement.

It should be mentioned that certain procedures may lead to distortion and uncertainty in the calculation of capital requirement at several points. At the same time, we believe that the benefits from the use of the methods (more prudent and stable calculation of capital requirement) outweigh the costs arising from the procedure (the extent of error and uncertainty related to the result), which means that the risk of the underestimation of the capital requirement is avoided in improving economic conditions.

Basically, the study examines four methods - the calibration-based method, the TTC (Through-the-Cycle) rating-based method, a method based on the Vasicek model and the PRA methodology (two of these (the Vasicek and the PRA¹ methods are described in detail), which more or less ensure the stability and all-time prudent level of capital requirement, including improving economic conditions, as well.

2. ABOUT PROCYCLICALITY

It already became obvious in connection with the introduction of the standard method that the capital requirement calculation method itself is procyclical. The reason for this is that, during economic downturn, the capital requirement of institutions does not change or only slightly changes (decreases). In contrast, solvency capital, by which the institutions cover their capital requirements, decreases capital adequacy to a larger extent owing to a negative result. Based on the above, the institution is urged to either involve resources (diverting resources from other economic operators at the same time), or reduce its balance sheet (e.g.

¹ PRA: the Prudential Regulation Authority is the prudential supervisory institution of the Bank of England

by not replacing expired loans) to an extent that the decreased solvency capital should still cover the risk of the loans which still remain on the balance sheet. Both actions (the involvement of resources or the reduction of lending) worsen the state of the economy, make recovery from the economic downturn more difficult or even aggravate the economic crisis.

Compared to the standard method, the IRB approach also uses the risk parameters related to the exposures, and can lead to the volatility of capital requirement primarily through the change in PD. In view of the above, if the institution uses PDs which follow the economic cycle, the effect of the PDs, which are volatile due to the cycle, makes itself felt in the capital requirement. In the event of an economic recession, compared to the standard method, a stronger procyclical effect is expected in this case, as the capital requirement of the institution will increase owing to increased PDs, while its solvency capital decreases at the same rate as in the previous case, therefore the gap between the capital requirement and the solvency capital that covers it increases (as, in the case of the application of the standard method, the capital requirement remains almost unchanged). Based on the above, the institution is forced to intervene more intensely, raise capital or reduce its balance sheet.

Figure 1



Capital requirement² depending on PD

Source: own editing

As Figure 1 shows, in the relevant (low PD range), capital requirement is monotonically increasing depending on PD, therefore the increased PDs also mean higher capital requirement.³

² In our article, the terms "capital requirement" and "unexpected loss" are used as synonyms.

³ IRB capital function for retail mortgage

Nor is procyclicality occurring in relation to capital adequacy desirable from the point of view of supervision.

In the event of economic boom, due to falling PD levels, a unit of exposure can be financed and covered by solvency capital more cheaply by the institution than during an economic crisis, which results in growth in the institution's balance sheet.⁴ In this case, the regulator has prudential concerns regarding decreased PD levels which, through IRB determine a lower capital requirement that fails to sufficiently cover unexpected losses.

2.1. The terms Point-in-Time (PiT) and Through-the-Cycle (TTC)

Bank rating systems are typically hybrid systems, i.e. they are between the theoretical Point-in-Time (PiT) and the Through-the-Cycle (TTC) rating systems. This means that they have not exclusively PiT or TTC features, but rather a mix of them. In order to avoid misunderstandings, in this chapter, we clarify what we mean by PiT and TTC rating systems. Professional literature does not use these terms consistently, we can meet different TTC concepts. In the Vasicek model (Vasicek (2002), on which the IRB is based, rating philosophies do not even appear. We can meet the terms conditional PD (PD in a given state of the economy) and unconditional PD instead. Professional literature often mentions them as PiT PD and TTC PD (and as estimates), which are not easy to estimate and place in Vasicek's framework system. Furthermore, it is not obvious either, how TTC PD can be determined based on the PiT rating systems and how it differs from PD derived from a TTC rating.

The ratings given by rating institutions (Moody's and S&P) are often considered to be TTC ratings. These are more stable over time than e.g. PiT ratings provided by KMV⁵ (the same can be observed in the case of several domestic institutions). Based on their internal documentation (*Gordy*, 2006), rating institutions strive to filter out the effect of the change in the economic cycle from client rating. According to *Carey* and *Hrycay* (2001), TTC ratings issued by rating institutions take into account the probability of the situation in which the client survives a stress scenario. Even in this case, as the stress scenario is fix, rating is independent of the

⁴ The article does not deal with the methodology and application of countercyclical capital buffer.

⁵ The KMV model is a structural model that determines the credit risk of a given company depending on its asset-liability structure. The model calculates the probability of default by extending the Black-Scholes-Merton framework system. Among other things, KMV set up a default database, by means of which the distance to default values calculated in the model are linked to the empirically observed probabilities of default. An important feature of the calculation of distance to default is that not only determined by the level of liabilities, but the distribution of short- and long-term liabilities was considered, as well.

current state of the economy. As *Cantor* (2001) writes, when assigning ratings to clients, Moody's minimises sudden migration between rating categories. Rerating occurs only when a client is unlikely to get back to previous rating category within a short period.

Based on the material published by the PRA (Bank of England, 2015), PiT and TTC rating systems can be defined along the following features.

PiT system:

- estimates default risk within a fix, typically 1-year period;
- in a PiT rating system, the increase of default risk usually entails migration into worse rating categories;
- default rates in each rating category are more stable, and are closer to the PD of the category;
- leads to volatile capital requirement in time.

TTC system:

- the institution seeks to filter out volatility caused by economic cycles from default risk and measure the client's risk throughout the cycle;
- the TTC rating does not react to the changes in the economic cycle, therefore the capital requirement is not volatile (merely due to changes in the economic cycle);
- the current default rates are volatile in the individual rating categories (their movement follows the cycle fall during economic boom, and rise during recession), and differ from the PD of the category;
- leads to more stable capital requirement in time.

It is important that the rating philosophy (to what extent is it PiT or TTC) should not be mixed up with the expectation according to CRR which prescribes that PD estimation shall be based on the long-term average of default rates by rating category. Merely calibrating PDs to the long-term average of default rates in each category will not turn any PD to a PiT or TTC PD.

BCBS (2016), which is under market consultation at the moment, suggests that rating systems should be established in a way that ensures the stability of the rating categories in time, through the economic cycle. Migration from one category to another should occur only in the case of idiosyncratic or industry-specific changes, and not owing to the change of the economic cycle. The idea above is also in line with the PRA definition. In the material, the terms TTC rating and TTC PD are used in this sense.

Despite the relevant rules on the current PD (Basel, CRR, EBA Guideline), the current regulation is not so uniform and fully-fledged. It does not define concretely which PD is the input of the IRB capital function. As a result, we can find PD models with different capital requirement levels in all ranges of the PIT – TTC scale all over the world.

As the figure with the map below shows, in North America, basically, the PiT PD values are adjusted to a long-term average in the course of capital calculation. In the United Kingdom, probability of default is scaled up to through-the-cycle level by the scalar approach detailed in the article, while in continental Europe, the conversion of parameters, which were estimated by means of short-term PiT models instead of TTC models, to long-term values was the trend. In Australia and New Zealand, structural models are spreading, while in the eastern regions of Asia, the PiT approach is dominant.

Figure 2 PDs used for the calculation of capital requirements



Source: own editing, Ben Begin (2012)

3. METHODS FOR REDUCING PROCYCLICALITY

As we mentioned in the introduction, capital regulation and the procyclicality of the behaviour of banks have undesirable consequences. Based on Basel III, the reduction of the aforementioned two factors is a high priority goal. On the other hand, it is also true that Basel II also intended to subdue procyclicality by requiring downturn LGD and with PD estimation reflecting long-term experience. Basel III specifies several methods for handling procyclicality. Capital buffer built up above the minimum level can be used in the event of incidental stress, as well. A means of this is the capital maintenance buffer requirement, which is complemented by countercyclical capital buffer that is to prevent unsustainable capital expansion in the period of overheating of the financial system. Basel III also declares that the procyclicality in the minimum capital requirement shall be reduced, as well. In addition, Basel III also acknowledges that risk sensitivity and procyclicality are inseparable from each other to some extent.

Gordy (2006) mentions three methodologies for decreasing the procyclicality of the capital requirement:

- i. The use of the TTC rating system, in which the rating of the client does not include the effect of the economic cycle. If PD is calculated from the average of long-term default rates by rating category in such a system, the TTC system reduces the sensitivity of the client's PD to the macroeconomic environment.
- ii. In order to reduce the sensitivity of the capital requirement to PD, the capital function itself can be smoothed, as well.
- iii. The third option is when the result of the IRB capital requirement can be flattened out.

Point i. is obvious, we would not like to comment on it.

In connection with point ii., we note that in non-retail case, in the course of the application of the IRB capital function, the asset correlation is the decreasing monotonic function of PD. Due to this observation, smaller companies have higher PD and lower asset correlation than large companies. The incorporation of this connection into the IRB reduces the procyclicality of IRB, as higher PD would result in higher capital requirement, which would be subdued by lower asset correlation belonging to the higher PD.

In point iii, we will give a short summary of one of the (lesser-known) procedures described by Gordy (2006), which smooths the regulatory capital requirement received as a result, instead of risk parameters. The authors call this procedure "countercyclical indexing". The essence of the method is that the regulator determines an α multiplier for each period. The smoothed value is calculated by multiplying the capital requirement calculated from the IRB formula by this multiplier.

$$\hat{C}_{i,t} = \alpha_t C_{i,t} \tag{1}$$

The essence of the approach is that the change of the procyclical PiT PD within the cycle and the consequent rise or fall in capital requirement can be decreased by means of the α parameter. The α value can be calculated form the exponential weighting of the states of a macroeconomic factor at different points in time and by means of a correction parameter calculated from the variance of α . The α value uniformly applies to all institutions supervised by the given regulator. *C* refers to institution *i*'s capital requirement at a given time (*t*.), while \hat{C} refers to the smoothed capital requirement (with the α parameter) concerning the same institution at the same time. We should note that this methodology is similar to the countercyclical capital buffer, though the fundamental aim of the latter one is to subdue the overheatedness of lending and slow down excessive credit growth. The problem with the approach outlined above is that it ignores the fact that the capital requirements of individual banks move cyclically with the state of the economy differently, therefore the determination of a uniform α parameter does not lead to countercyclical capital requirements in the case of all banks.

In the course of calculating the capital requirement that is stable over time, we tested several methodologies, including calibration methods (calibration of the development sample to long-term average), the practice of PRA (variable scalar approach), the calculation of PD based on TTC rating and the approach based on Vasicek's model (where unconditional PD is the expected value of conditional PD). We describe the PRA approach and the approach based on the Vasicek model in detail, because the use of calibration methods is well-known among banks. However, we will comment on the latter and present experience based on TTC rating.

4. POSSIBILITIES OF CALCULATING TTC PD

The possibilities have been examined related to two main problems:

- It is necessary to determine the default rate at the level of the long-term portfolio if the time series of an institution is not long enough. The long-term average of default rates is a necessary input for calculating capital requirement that is stable over time, because, as we have seen, the IRB formula applies a PD that is calibrated to a target level which includes worse economic years, as well.
- 2) Calculating TTC PD for a current bank portfolio.

The primary goal, which connects the two problems above, is to subdue the procyclicality of the capital requirement and create its stability. First, out of the methods suggested by Gordy (2006) to decrease procyclicality, we examined those which achieve the desired goal by calculating PD.

As the first problem, we examined by what method an institution whose default rate time series is not long enough could reproduce the default rate time series. In view of the above, a link should be established between the default rate time series of a given institution for a specific portfolio and the macroeconomic variables, then, being aware of the change of the macroeconomic explanatory variables, we estimate the past default rates retroactively so that the length of the time series will cover a whole cycle. The meaning of economic cycle has not been defined yet, therefore it is still unclear the actual or estimated default rates of how many years should be considered. The PRA recommendation could be a good starting point, because it suggests that periods of economic boom and recession should be equally represented in the time series. The data series from the past 10-12 years could be an appropriate choice for the description of a cycle.⁶

The relation between the default rate and the macroeconomic explanatory variables should be established for each institution (each important segment) individually.⁷ The main reason for this is that the institutions define default differently.

The second problem is the calculation of TTC PD at the level of the client / transaction so that it can be integrated into the risk-sensitive capital requirement calculating engine of IRB or the bank.

The institutions often calibrate the PD of their development sample to a long-term average default rate. In relation to this, based on the experience of supervisory Internal Capital Adequacy Assessment tests (mainly in the case of corporate PD models), it has been observed that if the applied variables follow the movement of the economic cycle, the PDs for individual years are volatile, as well. Although the calibrated PD of the whole development sample will equal the long-term average of default rates (central tendency), the PDs, following the movement of the cycle, calculated for individual years will sometime be below average, and sometimes above average. The extent to which PDs for individual years differ from the longterm average of the default rate primarily depends on the proportion of variables which follow the movement of the economic cycle and on how sensitive these variables are to the economic cycle.

First, we will discuss the second problem, supposing that the default rate time series of the institution is long enough, and then we will deal with the first problem. In this case, the existing default rate time series will be extended, and the default rate data of the extended time series will be used for calculating TTC PD.

4.1. TTC rating

In addition to converting the applied PDs to TTCs (see: subsequent models), there is another opportunity: when the rating system itself is a TTC type system, and the related PDs are the input of the IRB capital function. The TTC rating of clients ensures that clients get into another rating category only if their individual

⁶ For those who read this article 5-6 years later, the aforementioned period of 10-12 years will not necessarily be the appropriate choice.

⁷ This could be a problem if certain methods are applied, for example, the institution would separate effects arising from systematic and individual shocks. Of course, the systematic effect can be better measured at the level of the banking system that at the level of individual institutions.

risks change. Gordy (2006) calculates the average default rates belonging to the individual TTC ratings through the cycle, and allocates them to rating classes. It is the most achievable in the retail segment, as the application ratings used by the institutions in this segment typically include variables which are largely independent of economic cycles (socio-demographic variables, mainly PTI), i.e. they are neither TTC-, nor PiT-type variables. The institution has to run its current rating model back in time and classify its clients. After this rating, the average default rate observed in each rating category can be the basis of a TTC-type PD (which is generated, for example, in a logit model).

Another advantage of this rating is that it also indicates the change in the quality of the portfolio, therefore it is observable irrespective of cycle how the percentage of clients with a better rating changes (e.g. due to changing lending policy). In the case of a credit institution applying PiT PD, PDs can improve even if the portfolio is deteriorating if the state of the economy is improving at a higher rate than the rate at which the portfolio is getting riskier.

Another option is when the institution builds a PD model on a sample which covers a long cycle in time by using application variables (e.g. by logistic regression), then uses the results for its current portfolio.

4.2. PRA and the variable scalar approach

The PRA uses the so-called "variable scalar" methodology to convert point-intime parameters into cycle-independent parameters. The main point of the procedure is that, under certain conditions, by multiplying the point-in-time estimate, which follows the movement of the cycle in time, by a scalar value, which also changes in time, the probability of default of the portfolio is adjusted to the longterm average at portfolio-level. The scalar value is calculated at portfolio-level, i.e. the average of the portfolio-level long-term rates is divided by the current portfolio-level default rate, the PiTs and PDs in each rating category are multiplied by this scalar value.

Consequently, the PDs by category will be different, but the average PD of the portfolio will equal the long-term average. Furthermore, the portfolio-level PD always equals the long-term average, which, as a result, fails to manage structural changes inherent in the time series. Let us suppose that the Bank has PD1-PD7 performing categories, to which it assigns PiT PD.

The quotient of the average of the long-term default rate and the current default rate shall be k1. In this case, the institution multiplies the PiT PDs by k_1 scalar in order to calculate the capital requirement. After multiplication, the average of the default rate will be exactly equal to the portfolio-level PD. At the same time, if the

bank decides to let in only those clients who belong to the PD1 category, current default rates will fall (as the portfolio is better), while the long-term default rate will not or just slightly decrease, depending on the length of the time series.

In this case, (if the average remains the same, for sure) the new scalar $k_2 > k_1$ will be realised, which means that regarding PD, the bank draws the higher quality portfolio to the long-term average, therefore, the "same" capital requirement will be assigned to it as to the previous portfolio of poorer quality.⁸

This problem (change in the quality of the portfolio) is not the same as when the rating of clients changes in the PiT rating system owing to economic cycles, and as a result, the portfolio-level default rate fluctuates. For example, in Hungary, the rating of clients has shifted in the right direction due to economic boom. The main aim of the PRA's methodology is to tackle this latter problem, while the first problem is related to the change in the quality of the portfolio, which the methodology does not manage indeed.





Source: own editing, Moody's Analytics

It is important to emphasise that instead of calculating the TTC PD, the aim of the PRA regulation is to ensure the stability of the capital requirement. The PDs applied by the institution are converted by means of the variable scalar approach so that the PD integrated into the IRB capital function should be stable (on average), more or less ensuring the stability of the capital requirement itself.

Concerning the calculation of long-term average values, the PRA of the Bank of England expects the institutions to segment their portfolios according to the

⁸ Of course, it is not completely true, as the distribution of EAD among rating categories in the IRB is also important. On the other hand, the smoothing of capital requirement in time is achieved.

anticipated effect of the main controls of the underlying risk factors on default. For example, in the case of mortgages, the ability and willingness to pay, which are mentioned by the DTI (debt-to-income) and the LTV (loan-to-value) indicators, can be mentioned. For each pool created in the manner described above, the institutions shall calculate long-term average default rates based on the data of a whole cycle. The regulatory authority requires that the periods of economic boom and recession should both be represented in the estimation sample.

In the case of so-called "low default" portfolios, i.e. portfolios with few default events in a given period, PD should be estimated by a statistical method, the fitting of a distribution, as here the application of the raw default rate would be pointless. For the sake of conservative estimation, referring to CRR⁹, the PRA expects probabilities of default estimated on the basis of the upper part of the confidence interval.

In the course of the calibration of the models, the PRA seeks to limit cyclicality, thus reducing the effect of cyclicality. Consequently, the institutions should take into account the 30% upper limit on cyclicality when adjusting the default rates to long-term average for the years in which no observable default event occurred at the given rating level. Based on this, even in the case of a 0% default rate, the PD has to be calibrated in a way that brings it close enough to the long-term average. This 30% maximum cannot be applied if, in the course of calibration, most of the data series of the default rate derives from the downturn period. By this procedure, the instability of the long-term PD can be decreased in accordance with the requirements of the PRA.

The PRA quantifies cyclicality with the following formula¹⁰:

Cyclicality (%) =
$$100 \left(\frac{PD_t - CT}{DR_t - CT} \right)$$
 (2)

The formula is simply the ratio between the changes of the estimated and the observed values, where PD_t refers to the long-term average probability of default at a given moment in time, while DR_t means the default rate at the same moment in time. CT equals the level of the average default rate measured in the cycle. As the formula shows, the more stable the values of probabilities of default are and the closer the default rates are to the long-term average, the model contains the less cyclicality. Thinking backwards, by maximising the above-mentioned ratio at 30%, probability of default, and thus the volatility of the capital requirement, can be reduced in the course of the model calibration. Based on the ratio above, it is clear that the less the difference between CT as average default rate and the

⁹ Capital Requirement Regulation

¹⁰ Other indices can be defined for the measurement of cyclicality, as well: e.g. see: PETROV-CARLEHED (2012).

long-term average *PD* is, the smaller the numerator and the percentage cyclicality indicator are.

Based on the observations of the PRA, in the case of most portfolios, the institutions find it difficult to separate explanatory variables which are cyclical or independent of cycles in the course of modelling, therefore, capital requirement can show undesirable fluctuations in many cases. For example, models with excessive point-in-time character can lead to estimates which are considerably below the long-term average in periods of economic boom. The aforementioned assumption about cyclicality with an upper limit aims to subdue these effects. This is especially true in the case of the mortgage portfolio, which is one of the most crucial portfolio segments in the Hungarian banking system, as well.

In the period of economic boom, the PRA does not allow the institutions to return to point-in-time estimation. Therefore, the institutions have to act consistently in the course of estimations if they wish to determine their capital requirements based on parameter estimates independent of the cycle.

When analysing the variable scalar approach, we encountered the problem that the stable capital requirement does not necessarily follow from the PRA methodology. The reason for this is that, on the one hand, multiplication by the scalar value ensures that the average of the PDs used for the calculation of capital requirement (i.e. the portfolio-level PD) equals the long-term default rate, on the other hand, the PiT character of the rating system determines the rating of exposures. In a PiT rating system, exposures change with the cycle, therefore the exposure itself changes category, as well. However, the IRB capital function assigns different capital requirements to different rating distribution, even if the portfolio-level PDs are the same. The reason for this is that the relationship between PD values and the capital requirement is not linear in the IRB capital function, therefore, in addition to the PD level, the function is also sensitive to the dispersion of PD values.

In the following imaginary example, we converted the average PiT PDs of rating categories to TTC PD values by applying the variable scalar approach. For the sake of simplicity, we set a 40% value, which can be considered downturn, for each category. In accordance with the Basel recommendation, we set 15% as the value of asset correlation. Within the mortgage portfolio, migration between rating categories is the same in both cases. In order to be able to measure the effect of the differences between PDs more clearly, as a simplifying condition, the portfolio EaD should be the same through the whole 3-year path (static portfolio). Besides, each exposure was considered to be a unit, therefore weighting on the basis of exposure and the number of items leads to the same result. The portfolio is granular. In the first case, the original PDs by category are substituted into the IRB formula, while in the case of the "scalar approach", the average portfolio-level PD multiplied by the scalar value variable in time corresponds exactly to the target level derivable from the long-term average. By inserting the PD values transformed to the received TTC level, the risk-weighted assets value (RWA) can be calculated. The table below and Figure 4 show the change of the capital requirement along the paths of the "Point-in-Time approach" and the "Scalar approach".

Table 1

An example illustrating that the variable scalar approach only reduces the volatility of the capital requirement, but does not put an end to it

Point-in-Tim	ie									
Rating	Туре	PiT PD (1)	PiT PD (2)	PiT PD (3)	EAD 1	EAD 2	EAD 3	Cap.Req.	1Cap.Req.	2Cap.Req. 3
1	Mortgage	1,00%	1,00%	1,00%	100	100	0	4,25	4,25	0,00
2	Mortgage	2,00%	2,00%	2,00%	100	50	50	6,63	3,31	3,31
3	Mortgage	5,00%	5,00%	5,00%	100	150	150	11,17	16,76	16,76
4	Mortgage	8,00%	8,00%	8,00%	100	50	50	14,02	7,01	7,01
5	Mortgage	13,00%	13,00%	13,00%	100	100	100	16,98	16,98	16,98
6	Mortgage	15,00%	15,00%	15,00%	100	150	150	17,77	26,65	26,65
7	Mortgage	18,00%	18,00%	18,00%	100	100	200	18,65	18,65	37,31
Average		8,86%	9,57%	12,00%				12,78	13,37	15,43
Sum								89,47	93,62	108,02
Change										
Sum								18,55		
% Growth								+20,73%	1	
Scalar Appro	ach									
Rating	Туре	PD TTC Scalar 1	PD TTC Scalar 2	PD TTC Scalar 3	EAD	EAD 2	EAD 3	Cap.Req.	1Cap.Req.	2Cap.Req. 3
1	Mortgage	1,15%	1,06%	0,85%	100	100	0	4,65	4,42	0,00
2	Mortgage	2,29%	2,12%	1,69%	100	50	50	7,20	3,43	2,99
3	Mortgage	5,73%	5,30%	4,23%	100	150	150	11,97	17,27	15,33
4	Mortgage	9,16%	8,48%	6,76%	100	50	50	14,86	7,19	6,49
5	Mortgage	14,89%	13,78%	10,99%	100	100	100	17,73	17,31	15,98
6	Mortgage	17,18%	15,90%	12,68%	100	150	150	18,44	27,10	25,25
7	Mortgage	20,61%	19,07%	15,21%	100	100	200	19,19	18,90	35,69
Average		10,14%	10,14%	10,14%				13,43	13,66	14,53
Sum								94,04	95,61	101,73
Change										
Sum								7,69		
% Growth								+8,18%		
Long term average		10,14%								
Scalar		1,15	1,06	0,85						

Source: own editing

In the example above, applying the PRA methodology to a static portfolio, we face two effects:

- i. During economic downturn, portfolio-level default rates also rise, which decreases the scalar value. As a result, the portfolio-level PD remains the long-term average of default rates.
- ii. Due to the PiT rating system, there were significant changes in the portfolio. The distance between the individual rating categories can be significant (basically, there is little difference in PDs between the good categories of Masterscale, while in the case of bad categories, the difference is large). Even if the scalar value falls during recession, the (rescaled) PD belonging to an exposure in a worse category is likely to be higher than in the original category of the exposure, which results in higher capital requirement.

On the whole, we believe that, although the variable scalar approach does not fully ensure the stability of the capital requirement (see: the resultant of the two effects above), it really provides a more stable capital requirement than as if the institution used the original PiT PDs (without the scalar value) to calculate the capital requirement. Stable capital requirement can only be partly achieved by the calibration of PDs.





Source: own editing

In view of the above, the rescaling of PDs belonging to rating categories in a PiT rating system in the manner described above ensures neither the full stability of

the capital requirement, nor that the applied PDs represent the long-term probability of default of exposures (clients/transactions). In banking practice, the PiT rating system is usually realised by the so-called behavioural scoring, which uses the characteristics of the current liquidity of the exposure, e.g. information on delay. Behavioural scoring can predict default well on the 1-year horizon relevant to capital calculation, therefore it is characterised by a high-level of separating ability.

As a result of the high-level of separating ability, rating categories encompass a wide PD range. The rating system classifies the currently non-problematic exposures to a PD category much lower than the long-term average, while the currently problematic exposures with payment difficulties to a PD category much higher than the long-term average. In order to assure that PD reflects average long-term probability of default, in a behavioural / PiT rating system rescaling should not take place evenly, but the PD of exposures better than the average PD should be rescaled upward, while the PD of exposures worse than the average should be rescaled downward. Although this procedure would lead to the same average PD at portfolio-level, regarding capital requirement calculation according to IRB, it would give a result that is completely different from that calculated by the scalar approach, by rescaling PDs with the same factor.

Based on the criticism presented above, the scalar approach does not lead to either a PD based on TTC rating in the case of individual exposures, or a capital requirement completely independent of the cycle, therefore its use cannot be considered as an extensive solution for the problems described in this article. In reality, the only advantage of the scalar approach over TTC rating is that it is easy to implement.

4.3. A short description of the Vasicek model

Among other things, Vasicek's article (2002) on the change of the value of the loan portfolio revealed that in the single-factor model, which is also the basis of the IRB capital function, under certain model conditions, PD is distributed in accordance with the third equation below:

$$p(Y) = P[L_i = 1|Y] = \Phi\left(\frac{\Phi^{-1}(p) - Y \times \sqrt{\rho}}{\sqrt{1 - \rho}}\right)$$
(3)

where p(Y) refers to the conditional PD of the client/transaction, i.e. the PD in a given state of the economy. Φ is the cumulative distribution function of standard normal distribution, ρ is asset correlation, Y the underlying macroeconomic factor. We still cannot find the terms PiT and TTC. Conditional PD and unconditional PD are mentioned instead. Here, we refer back to Gordy's (2006) simula-

tion procedure, in which the average of the PiT PDs belonging to TTC ratings was applied in the capital function in the course of the simulation of the capital requirement. In Vasicek's (2002) model, p in the formula above refers to unconditional PD, the average of conditional PDs belonging to the individual scenarios (different states of the economy).

According to the material of the PRA (Bank of England, 2015b), the unconditional PD of the Vasicek model definitely corresponds to TTC PD, therefore the PRA TTC PD definition applies to it.

The formula above applies to client-level. The unconditional probability of default in the Vasicek model is the average of the client's PiT PDs in time. This may be the main reason for which this model differs from both the CRR and the banking practice. Averaging the default rates of clients in a given rating category is not the same as averaging the individual PiT PDs of a client in different states of the economy.

Averaging the default rates by rating category may be a good estimation method for the input of the Vasicek model and the IRB only when it is conducted in a TTC rating system, i.e. the rating of clients does not change under the influence of economic changes. In this case, averaging the default rates of the rating category, the unconditional PD of the Vasicek model is estimates as a possible estimate of PiT PDs.

In the case of the PiT rating system, the correct input can be calculated for the model by averaging the PiT PDs by client, irrespective of the rating categories of the client. At the same time, if the average of the category is determined in each rating category, clients move to other categories from time to time, therefore are averaged only with currently similar clients, which only minimally subdues the fluctuation of capital requirement. A given category has always included clients who fitted into that category, therefore the average default rate will be very close to the PiT PDs at any time.

In the case of the PiT rating system, the unconditional portfolio PD, which serves as the input for the Vasicek formula can be calculated by averaging the portfoliolevel default rates. Vasicek (2002) uses the same portfolio-level PD as in the case of individual clients. In short, the publication itself does not provide specific guidance for everyday use.

Schaefer (2012) describes the terms of the Vasicek model as follows:

- homogeneous portfolio;
- a large number of loans.

where homogeneous portfolio shall be construed as follows:

- the same probability of default (*p*);
- (implicitly) the same LGD;
- the same asset correlation.

There is an opportunity to calculate the unconditional PD if the formula above includes both the conditional and the unconditional PDs. The next chapter will describe this calculation in context, but the result is presented here in advance. The formula is based on the dispersion (σ) and expected value (μ) of default rates.

$$\widehat{PD_{TTC}} = \phi\left(\frac{\hat{\mu}}{\sqrt{1+\widehat{\sigma^2}}}\right) = \phi\left(\frac{\frac{1}{m} \times \sum_{i=1}^m \phi^{-1}(DR_i)}{\sqrt{1+\frac{1}{m} \times \sum_{i=1}^m \phi^{-1}(DR_i)^2 - (\frac{1}{m} \times \sum_{i=1}^m \phi^{-1}(DR_i))^2}}\right) (4)$$

It is observable that, if our time series is long enough, the unconditional PD can be calculation through the expected value and dispersion of default rates.

Problems related to estimation will be discussed later. However, we would like to note that, as it is a single-factor model, the PD estimate will be different from the on ewe would expect in the case of independent defaults. We also mention that the estimation of unconditional PD cannot be separated from the estimation of asset correlation (ρ), therefore the two change together in a consistent estimate. The breakdown of the portfolio-level PD at client level is detailed in the next chapter.

5. TTC PD ESTIMATION, COMPLETION OF THE TIME SERIES

In the case of the calculation of the TTC (i.e. Through-the-Cycle) PD, the longer a time series is, the more likely it is to cover an economic cycle. As we mentioned earlier, in most cases, 10-12 years could be enough to provide an appropriate "estimate for the probability of default that is independent of cycle. However, if the time series is not long enough, completing the time series retroactively or in advance could be an appropriate solution. In order to do so, we can heavily rely on the relation between the number and rate of macroeconomic variables and defaults changing with the business cycle.

The relationship between volatile default rates in the cycles, the TTC PD, which is mainly independent of the cycle, and the economic factors characterising the cycle is detailed below. As an introduction, it is worth mentioning that currently, underlying economic processes, irrespective of whether they are selected into the model individually or a common underlying main component is estimated, are most frequently used to provide a historically available, objective description of an economic cycle. In our view, including macroeconomic variables into the model is an appropriate solution for establishing the connection between the change of the business cycle and that of default rates sensitive to it.

¹¹ Instead of the degree of the estimation error, appropriateness refers to the length of the cycle.

The following figure shows the systematic risk of the mortgage portfolio of a given bank calculated on the basis of a 10-year-long time series (*Y*, see: equations 6 and 9). The following figure shows the change of the systematic component. It is clearly visible that the systematic component almost covers a whole economic cycle.

Figure 5 Breakdown of a default rate time series into systematic and individual components



Source: own editing

Figure 5 illustrates the separation of the systematic and individual components based on the default rate time series of a bank. The high-level symmetry between the individual component and the systematic component ceases if the systematic risk is calculated on the basis of the time series of several banks which relating to the same segment (and in the case of using PD instead of the default rate). Here, we should mention that in our opinion, the right approach is when estimation is based on several samples from banks. On the other hand, it is a problem that institutions have a different understanding of certain segments or do not have default rate time series for the same segments, which makes the application of the methodology more difficult.

A time series can be extended in different ways (see: the following points, i-ii), however, procedures are based on the estimation of relationships in short time series, and, on the basis of this, the provision of missing data. In our example, the PiT (Point-in-Time) PD corresponds to the default rate. The approaches are presented by means of the time series of the OPTEN default rate, as the estimation was conducted for a period related to which factual data existed, which was an advantage during the back-testing of the model.

i. According to the first approach, the default rate was directly modelled with macroeconomic explanatory variables. The time series of factual data was the average sectoral default rate between 2008 and 2016. The time series is available in the Stability Report of the National Bank of Hungary. Estimation should be conducted prudently, as the available raw time series is short and autocorrelated. In the light of this, the selection of methodology, keeping an eye on estimation errors, the application of error reduction procedures and sensitivity tests run for regression parameters are important.



Figure 6 The change of sectoral default rates between 2008 and 2016

Source: own editing, National Bank of Hungary Stability Report

The relationship was established by robust fitting. The advantage of this method is that it underweights data points which are farther from the mean value. Consequently, the more extreme values are, the less weight they have in the course of fitting (*Csereháti*, 2004). During this estimation, the unemployment rate deferred by 4 quarters and 1-month EURIBOR interest rate proved to be significant. Applying the detected relationship, we estimated the "artificial" sectoral default rate time series for the period between 2000 and 2008.¹²

As the period between 2008 and 2016 included an economic crisis, i.e. a downturn data series, the smoothing and underweighting of extremes values observes in

¹² The level of the EURIBOR 1M did not turn negative in the examined period, but since the end of 2015, the one-month short-term interests (e.g. EURIBOR) fell below zero on many occasions. In this case, expert correction may be needed in order to avoid the underestimation of the default rate, as regression might claim falsely that negative short-term interests decrease the probability of default proportionately, therefore the use of a floor value might be justified.

the course of robust fitting proved to be useful when the time series was being completed into a cycle. During the retroactive estimation and completion carried out for the 2000–2008 period of economic boom, we avoided significant overestimation and hecticity, therefore, from economic point of view, we received a reasonable default rate time series during back-testing. However, it is important to mention that retroactively, we would not have been able to estimate the downturn data series as efficiently by means of a relation that is supposed to have been significantly adjusted to the "smoothed" mean value. In other words, the supplementary crisis data series estimated by robust fitting would probably have provided less "downturn" results than the factual data.

The residuals received during fitting are autocorrelated, therefore we tested further model form, which are not detailed in this document. In addition, we examined the estimation error (and its effect on the capital requirement), which we found acceptable. At the same time, the above-mentioned problems should be continuously dealt with. It is important to emphasise that the estimation error is seriously affected by the length of the period that is available or that we wish to estimate, the size of samples and their relations with each other (Tarashev, 2009). Figure 7 below illustrates the distribution of the long-term default rate, which can be defined as follows: Based on the available time series (2005-2016), we established the link between macro variables (and their transformed versions) and the default rate. By means of this, we estimated the default rates for the period between 2000 and 2004, and calculated the long-term average default rate from the extended time series. Of course, the estimation has errors. The distribution of individual parameters is known or can be well approximated. In order to have an idea of the uncertainty of the estimation, we extended the default rate time series for the period 2000–2004 in a different way: by simulating (10,000) random numbers based on the distribution of the estimated parameters, we received new parameters by means of which we conducted the extension of the default rate time series for 2000-2004 (Sahinler-Topuz, 2007). The values of individual default rates also affect the change of the long-term average default rate, the distribution of which is shown in Figure 7. The figure includes the confidence intervals of 90% and 95% (the former is indicated by two dotted line, the latter by two continuous lines), which show that the simulated long-term default rates fluctuate within a narrow range, close to the factual data. The two lines in the centre of the figure refer to the real value of the long-term default rate (above the whole sample), as well as to the value received by means of the best estimate, which also support the proximity of the estimate and the factual data.





Source: own editing

The figure below illustrates the fitting of the original default rate values and the default rate values re-estimated from the short time series:

Figure 8 Default rate estimate for the period 2000-2008 based on the data from the period between 2008 and 2016



Source: own editing

ii. According to the other approach, we estimate systematic factor Y by means of macroeconomic explanatory variables. Based on this, we determined the supplementary default rate time series indirectly. Macroeconomic factor Y was also estimated by robust fitting for the same time series between 2008-2016. In this case, default rates were matched with PiT PDs values, as well. The relevance of the use of this method is that the assumptions of the model are in line with the IRB approach, according to which the default of individual transactions / clients is moved not only by the idiosyncratic factor, but also by a common systematic factor (Y). Due to its modelling, Y can be determined by several real economic factors.

By reorganising the formula below, we receive *Y*.

$$p_{Di}(Y) = PD_{PIT} = \phi\left(\frac{\phi^{-1}(PD_{TTC}) - \sqrt{\rho} \times Y}{\sqrt{1 - \rho}}\right)$$
(5)

$$Y = \frac{1}{\sqrt{\rho}} \times \left(\phi^{-1}(PD_{TTC}) - \phi^{-1}(PD_{PIT}) \times \sqrt{1-\rho}\right)$$
(6)

At the same time, in order to calculate it above the factual time series, we need to know the scale of TTC PD, PiT PDs and the asset correlation. As the first step of modelling, we estimate the asset correlation above the available (short) time series, the PiT PDs (which correspond to the default rate, Y_i continuously changes depending on the state of the global economy) and the TTC PD, which, regarding its content, corresponds to an unconditional probability of default, as it scale is (approximately) the same in the event of all Y_i states of the global economy.

For the estimation of asset correlation and the TTC PD, we use the following relation (based on the expected values and variances of default rates):

$$\hat{\rho} = \frac{\widehat{\sigma^2}}{1+\widehat{\sigma^2}} = \frac{\frac{1}{m} \times \sum_{i=1}^m \phi^{-1} (DR_i)^2 - (\frac{1}{m} \times \sum_{i=1}^m \phi^{-1} (DR_i))^2}{1 + \frac{1}{m} \times \sum_{i=1}^m \phi^{-1} (DR_i)^2 - (\frac{1}{m} \times \sum_{i=1}^m \phi^{-1} (DR_i))^2}$$
(7)

$$\widehat{PD_{TTC}} = \phi\left(\frac{\hat{\mu}}{\sqrt{1+\widehat{\sigma^2}}}\right) = \phi\left(\frac{\frac{1}{m} \times \sum_{i=1}^{m} \phi^{-1}(DR_i)}{\sqrt{1+\frac{1}{m} \times \sum_{i=1}^{m} \phi^{-1}(DR_i)^2 - (\frac{1}{m} \times \sum_{i=1}^{m} \phi^{-1}(DR_i))^2}}\right)$$
(8)

At this point, we distort the model by not calculating either asset correlation or the initial TTC PD for one cycle. Furthermore, this is point at which, instead of the time series of the given institution, the time series of the default rate typical of the industry (by segment) should be considered as a starting point, as *Y* means the systematic factor that uniformly affects all institutions of the banking system. Figure 9

There is the same systematic factor behind the observed default rates of individual banks, the difference between default rates is due to the qualitative difference of the portfolios



Source: own editing

The relation between the *Y* values, which were calculated from factual data, and the macroeconomic variables is described in the following form:

$$\hat{Y} = \alpha + \sum \beta_i \times macroeconomic \ variable_t \tag{9}$$

Based on equation 9, the value of Y is estimated for the periods which were unobserved in the original time series in a way that its values should cover a whole cycle. In principle, this estimation should be conducted only once for each homogeneous segment, possibly on the basis of the data from all banks. At the same time, the process is made difficult by the fact that the segmentation of the individual institutions is not the same.

Based on our experience we can state that, in order to create a composite default rate time series for all banks (i.e. representative for the given bank), we need the default rate time series of the banks, complemented by the EAD, along with the deepest possible segmentation (e.g. in the case of mortgage: FX, HUF, home equity, housing). Here, first of all, the advantages and disadvantages should be considered, because, if certain partial segments themselves contain sufficient default, no composite (segment-specific) time series is required. Otherwise, where we lose owing to the loss of homogeneity, the composite time series is required.

In the light of the above, this action (the estimation of *Y*) has to be taken by institution. For example, if a mortgage portfolio consists of HUF and foreign currencies, the rate of which continuously changed within the observed 10-year period, the default rate time series for all banks should reflect the same rate for appropriate estimation. In the case of this point, we did not examine the error that might arise if we calculated the value of *Y* by merely averaging the default rates of individual banks by segment. We note that the time series of individual banks are not of the same length, therefore, in the case of mortgage, we completed shorter time series on a pro rata basis.

By reinserting *Y* (the result of the *Y* estimate) into the original formula, we receive the extended, re-estimated default rates:

$$\widehat{DR_t} = \phi\left(\frac{\phi^{-1}(\widehat{PD_{TTC}}) - \sqrt{\widehat{\rho}} \times (\alpha + \sum \beta_i \times macroeconomic_{variable t})}{\sqrt{1 - \widehat{\rho}}}\right)$$
(10)

The fitting led to similar results as direct estimation (based on the relation between default rates and macroeconomic explanatory variables).

Figure 10

The change of factual default rates and their value estimated by means of the underlying macroeconomic factor between 2000 and 2008



Source: own editing

Based on the extended time series, we re-estimated the TTC PDs and the asset correlation. The whole time series of factual data for 2000-2016 and the TTC PD values complemented with factual data for 2008-2016, as well as with estimated data for the period 2000-2008 are close to each other according to both approaches. In the case of the first, direct default rate modelling, the average TTC PD calculated from factual data and the average TTC PD calculated from the "fitted" time series were 3.47/ and 3.56% respectively. In the case of the second, indirect estimate based on the modelling of macroeconomic factor *Y*, the TTC PD values calculated from factual data and the completed time series were 3.49% and 3.6% respectively.

The estimated and the factual data are close to each other, and the results of the two different estimates do not differ significantly. We examined the estimation error, as well. Although it was acceptable in this case, but a lot depends on the rate of the length of the available time series and that of the estimated period.

Based on the established portfolio-level TTC PD, the individual transaction-/client-level TTC PDs (q_i) should be calculated, and the capital requirement should be recalculated by means of these TTC PDs (see: *Petrov–Carlehed*, 2012). In addition to the institution's client-level PD, individual TTC PDs are determined by the value of *Y*. However, the PD of the bank is not exactly the PiT PD (p_i) mentioned in equation 11, but it is rather a hybrid probability of default $(p_{i,a})$. In view of the above, the PiT character (cyclicality) of the probability of default of the bank should be measured and considered in the course of the calculation of capital requirement.

While in the course of only PiT PD as follows:

$$q_i = \phi \left[\sqrt{\rho \times Y_t} + \sqrt{1 - \rho} \times \phi^{-1}(p_i) \right] \tag{11}$$

and in the case of hybrid PD, the client-level TTC PD of the Bank can be calculated with the formula below:

$$q_i = \phi \left[\sqrt{\rho \times} \alpha \times Y_t + \sqrt{1 - \rho \times \alpha^2} \times \phi^{-1}(p_{i,\alpha}) \right]$$
(12)

where α refers to the PiT character of the bank's PD¹³, Φ means the cumulated distribution function of standard normal distribution. The q_i value (client-level TTC PD) has to be used for capital requirement calculation.

The following figure shows the results run on the sample portfolio, along simplifications. It is important to note that, here, we also have to deal with the migration of EADs, the degree of which depends on the PiT character of the rating system.

¹³ α is a "PiT parameter", a real number between 0 and 1, the value of which is on a clearly TTC rating system, and 1 in a clearly PiT rating system. The calculation of α requires a risk profile of the portfolio that is stable in time. In order to ensure this a relatively stable portfolio-level average PD is required (Petrov – Carlehed [2012]). Refers to $\phi^{-1}(qi)$ the 11th equation (*Bi*), therefore the average of *Bi* individual values is expected to be stable in time. Calculating the following equation for several macroeconomic states of the global economy, then differentiating the portfolio averages, we receive an α value that subdues asset correlation: $p_{i,\alpha}(Y_t) = \phi \left(\frac{Bi-\sqrt{p \times \alpha \times Y_t}}{1-\alpha \times \alpha^2}\right)$.

Figure 11 Result run on a hypothetical portfolio averaged from the composite mortgage portfolios of several banks



Source: own editing

6. CONCLUSION

In our study, we tested four approaches - direct extension of time series, indirect extension of time series by the estimation of a macroeconomic factor, TTC rating system and the variable scalar approach applied by the PRA - for the purpose of ensuring the stability of capital requirement. each approach affected the IRB PD parameter. Along with the logic of the IRB function, the calculation of TTC PD based on the Vasicek model seems to be obvious, but the estimation of asset correlations is not an easy task. At the same time, we could observe that the TTC PD estimated on the basis of the Vasicek model was not far from the long-term average estimated with simple, linear regression (when the time series needed to be completed), even if the form of model used in the latter case is not quite compatible with the IBR. In the light of the above, it is not obvious whether it is worth choosing a more complicated model. Currently, PD calculation based on TTC rating seems to be easily feasible only in the case of the retail segment. As far as corporate portfolios are concerned, the feasibility of a rating system independent of cycle is not obvious. On the other hand, in this case, the scalar approach seems to be a better, more applicable method.

It is not clear either whether the regulator has to intervene on the input or on the output side to subdue procyclicality. If the state of the economy differs from the long-term average, the PD calibrated to the average does not comply with the aims of the bank anymore if the institution uses the PDs for purposes other than capital requirement calculation, as well. As if our watch did not work, we removed the battery and set it for 6 o'clock. It would show the exact time, while it would provide incorrect information in other cases.

We should not forget the costs of conversion to a TTC rating system either (Gordy, 2006). On the one hand, decreasing the volatility of the capital requirement is an obvious advantage. On the other hand, the TTC rating system is less suitable for active portfolio management (*Jarrow* et al., 1997). Furthermore, the published information (Pillar 3) provide much less support to the market in such a system, as the capital requirement follows the cycle less, therefore those who analyse the institution can find less information on capital requirement (BCBS, 2015).

Based on the above, on the side of the bank, if exclusively the TTC rating system were used, we would experience change in portfolio quality which would not be in line with the real, observable behaviour of the portfolio. At the same time, around the local extrema of the cycles, banks would considerably over- or underestimate the risk profile of their current assets. Just to mention an expressive example: as we can see in Figure 11, if we determined the need of the portfolio for generating impairment loss on the basis of the TTC PD, we would almost totally ignore the multiplicity of default events soaring in the crisis period between t11 and t31 in the course of the calculation of impairment loss.

At this point, it could be worth considering the smoothing of the capital requirement in time and publishing the smoothing procedure. In this case, the introduction of a PiT rating system would not hurt the interests of the institution, the market would receive the relevant information under Pillar 3 in three days (originally calculated, plus smoothed values), therefore, it could judge the institution regarding riskiness more easily.

REFERENCES

Bank Of England (2015): Supervisory Statement | SS11/13

- BCBS (2005): An explanatory note on the Basel II IRB risk weight functions. Basel Committee On Banking Supervision, Bank for International Settlements (BIS).
- BCBS (2015): *Revised Pillar 3 disclosure requirements* Basel Committee On Banking Supervision, Bank for International Settlements Standards.
- BCBS (2016): Reducing variation in credit risk-weighted assets constraints on the use of internal model approaches (issued for comment by 24 June 2016). Basel Committee On Banking Supervision, Bank for International Settlements, Consultative Document.
- BEGIN, BEN THOMAS, SUSIE (2012): Basel II retail modelling approaches PD Models. PwC lecture at Actuaries Institute.
- CANTOR, RICHARD (2001): Moody's investors service response to the consultative paper issued by the Basel Committee on Bank Supervision "A new capital adequacy framework". *Journal of Banking* & *Finance*, Elsevier, vol. 25(1), pp. 171-185, January.
- CSEREHÁTI, ZOLTÁN (2004): Az outlierek meghatározása és kezelése gazdaságstatisztikai felvételekben (The Determination and Management of Outliners in Economic Statistical Records). *Statisztikai Szemle*, 82. évf. 8. sz.
- EP (2012): Regulation (EU) No 575/2013 of the European Parliament and of the Council of 26 June 2013 on prudential requirements for credit institutions and investment firms and amending Regulation (EU) No. 648/2012.
- GORDY, MICHAEL B. HOWELLS, BRADLEY (2006): Prociclicality in Basel II: Can we treat the disease without killing the patient? *Journal of Financial Intermediation* 15, pp. 395–417.
- JARROW, ROBERT A. LANDO, DAVID TURNBULL, STUART M. (1997): A Markov Model for the Term Structure of Credit risk Spreads. *The Review of Financial Studies* 10(2).
- PETROV, ALEXANDER CARLEHED, MAGNUS (2012): A methodology for point-in-time-through-thecycle probability of default decomposition in risk classification systems. *Risk Magazine*, 6(3).
- SAHINLER, SUAT TOPUZ, DERVIS (2007): Bootstrap and jackknife resampling algorithms for estimation of regression parameters. *Journal of Applied Quantitative Methods*, 2(2).
- SCHAEFER, STEPHEN M. (2012): Vasicek's Model of Distribution of Losses in a Large, Homogeneous Portfolio. London Business School Credit Risk Elective Lecture, Summer.
- TARASHEV, NIKOLA A. (2009): Measuring portfolio credit risk correctly: why parameter uncertainty matters. BIS Working papers No. 280.
- VASICEK, ALFONS O. (2002): The Distribution of Loan Portfolio Value. Risk Magazine.