PROBLEMS IN THE APPLICATION OF CREDIT RISK MODELS[1]

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ABSTRACT
This paper presents the theoretical framework of the credit risk models applied by banks, the underlying statistical model and finally the most important problems encountered in their practical application. Firstly, the economic and the statistical models were originally designed for corporate portfolios. It is far from obvious how they should be adjusted in order to extend them to retail and SME portfolios. Secondly, it is highlighted that the widely-used Vasicek model refers to a single period, while a multi-period dynamic model would be more appropriate to the definition of, and the relationship between, through-the-cycle and point-in-time probability of default as well as to the characteristics of samples which cover different periods.

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1. THE ECONOMIC MODEL OF CAPITAL CALCULATION

For credit risk, the method generally applied and recognised by the different supervisory bodies for calculating the capital requirement is to match it with a downside risk measure, i.e. a defined percentile of the distribution of portfolio credit loss. This is usually described as value at risk, standing for the maximum credit loss expected at a defined confidence level which the capital of the credit institution should be able to absorb. That confidence level is 99.9% in the case of supervisory models. These models[2] are characterised by two main features:

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2 The legal formulation of the capital calculation model is Regulation (EU) No 575/2013. The document discusses the model and the conditions of its application in detail. The website of BIS contains numerous relevant guidelines and recommendations, part of which have been adopted and published on the website of MNB, the Hungarian supervisory authority. For an early theoretical foundation of the model see Gordy, M. (2003): 'A Risk-Factor Model Foundation for Rating-Based Bank Capital Rules', *Journal of Financial Intermediation*, 12.

a) The models employ additional assumptions to simplify calculations, especially the modelling of empirically well-established and strong correlations between default events. Such assumptions give rise to so called 'latent variable models', where

$$Loss(X) \rightarrow E[Loss|X]$$

Within these models, it is assumed that there is an underlying X – which describes the general state of the economy and is non-diversifiable – and if a portfolio is sufficiently diversified under the assumptions made, conditional expected shortfall given X provides a good approximation to the loss distribution. This is in line with the frequent occurrence in finance that, in the case of diversification, only non-diversifiable systemic risk matters.

Another assumption ensures that the 'value at risk' of the loss distribution (i.e. its quantile q) is the value it takes at the 'value at risk' of the random variable representing systemic risk[3] (i.e. quantile q of systemic variable x).

$$E\left[Loss_q|X\right] = E\left[Loss|X_q\right]$$

b) Loss distribution is estimated indirectly, by multiplying three variables:

$$Loss(X) = PD(X) \times LGD \times EAD,$$

where *PD* is the probability of default, *LGD* is loss given default and *EAD* is exposure at default[4]. It is further assumed that only the probability of default (*PD*) is dependent on systemic risk[5]. Therefore, continuing with the formula above

$$PD\left(X_q\right) \times \overline{LGD} \times \overline{EAD} \rightarrow Loss_q|X.$$

3 Consequently, extreme loss is loss arising in extreme economic situations. This may seem tautological only at first sight. It is dependent on the assumption that loss is a monotonically increasing function of the variable(s) describing the general state of the economy. Once a portfolio is constructed that is not based on such a straightforward relationship between loss and the state of the economy, that assumption is challenged.

4 *EAD* – unlike the two other measures – is not directly mentioned in the relevant legislation. It is defined as the product of the exposure amount and the credit conversion factor. However, the term *EAD* is more frequently used in the literature.

5 Examining the validity of that approximation is beyond the scope of this paper. It is disputable, for example, that the number of defaults is dependent on the macroeconomic situation within the model, but return is not. It is well-known that both *PD* and *LGD* are highly dependent on loan-to-value in the case of mortgage loans. Therefore, it is not very plausible that the former is sensitive to economic cycles while the latter is not. Nonetheless, this is the generally accepted calculation method prescribed by supervisory authorities as well.

Developing a capital calculation model is primarily about trying to define the three parameters above. It is mandatory for financial institutions to use the Vasicek model (see *Vasicek,* 1987) for determining conditional probability of default dependent on systemic risk.

Conditional probability of default in the Vasicek model is mainly based on *Merton*'s (1974) definition of default. According to this definition, a company defaults in an economic sense when the market value of its assets (A) falls below the nominal value of its external liabilities (B). In more general terms, default arises when the value of a random variable goes below a defined threshold.

In the Vasicek model, that random variable assumes a special form:

$$A_i = \sqrt{\rho} \times X + \sqrt{1-\rho} \times \varepsilon_i$$
$$X, \varepsilon_i \sim N(0,1).$$

where $N(.)$ is standard normal distribution. X generally stands for systemic risk, while ε_i are individual risks specific to company i. ρ is the correlation between any company i and j. As random variables have a standard normal distribution, so does A.

Based on the foregoing, the probability of default is as follows:

Default: $A_i < B_i \rightarrow PD_i = Prob(A_i < B_i) \rightarrow B_i = N^{-1}(PD_i)$

while conditional probability of default given X is:

$$PD_i(X) = Prob\left(\sqrt{\rho} \times X + \sqrt{1-\rho} \times \varepsilon_i \ < B_i\right) =$$

$$= Prob\left(\varepsilon_i \ < \frac{B_i - \sqrt{\rho} \times X}{\sqrt{1-\rho}}\right) =$$

$$= N\left(\frac{B_i - \sqrt{\rho} \times X}{\sqrt{1-\rho}}\right) =$$

$$= N\left(DD_i(X)\right) = N\left(\frac{N^{-1}(PD_i) - \sqrt{\rho} \times X}{\sqrt{1-\rho}}\right) \sim \textit{Vasicek distribution}^{6}$$

6 If a random variable can be expressed as $Y = N\left(\frac{B-\sqrt{\rho}\times X}{\sqrt{1-\rho}}\right)$, its distribution function is $F(y)=$ $=P(Y<y)=$ $P\left[N\left(\frac{B-\sqrt{\rho}\times X}{\sqrt{1-\rho}}\right) < y\right]$. By rearrangement and taking advantage of X following a normal distribution, we arrive at $F(y) = N\left(\frac{\sqrt{1-\rho}\times N^{-1}(y)-B}{\sqrt{\rho}}\right)$. This is the Vasicek distribution function. By differentiating this function the density function may also be obtained (TASCHE, P. 2008).

Based on this, conditional probability of default shows a two-parametric (PD_i, ρ) distribution, also known as the Vasicek distribution.

This model may be used for purposes other than the calculation of the capital requirement. The same formula $Loss(X)=PD(X)\times LGD\times EAD$ is used; only the interpretation of the respective parameters and thus the method of calculation will differ slightly. For instance, assuming that X can be described by a macroeconomic model, $X=w_0+\sum_i w_i\times f_i+\epsilon$, where f stands for different macroeconomic variables, it can be used for the analysis of different stress scenarios. In this case, $PD(X)$ is not an extreme value in the Vasicek distribution but a value derived from the macroeconomic model.

There are a number of questions arising in connection with this economic model, starting with the suitability of value-at-risk for the quantification of credit risk.

Or, how appropriate is the procedure proposed for the calculation of value-at-risk, considering either the approximation of the loss distribution by conditional expected shortfall or the calculation of loss as a product? Most regulatory authorities call for the validation of the presumptions of the model, including the so called granularity criterion. The granularity criterion ensures diversification, which is not met by most corporate portfolios. Splitting up loss into a product of $PD(X)\times LGD\times EAD$, where only PD is dependent on the state of the economy is also a criterion which regulatory authorities seek to validate by analysing the correlation between LGD and PD. This may lead to the establishment of an additional capital requirement; however, their definition is obviously outside the scope of the model.[7]

The extent to which Merton's definition of default is applicable to retail clients and how a better model could be constructed for this specific segment are also intriguing questions calling for further study.

2. STATISTICAL MODEL

As probabilities of default (PD) – both conditional and unconditional – are not directly observable, an estimation method should be devised. Probabilities of default are estimated through the observation of default rates (DR). In practical terms, the number of defaulting members within a cohort is determined[8]. The

7 Interestingly, the very first methodology proposed for capital calculation made an attempt at quantifying at least the granularity criterion based on the Hirschman-Herfindahl index of the portfolio. It was omitted from later recommendations.

8 For instance, the subject of analysis is the 2016 rate of default on mortgage loans disbursed in 2008. The result depends partly on the adopted definition of default. In the case of retail clients, the legislative provisions regard a delay of more than 90 days as default. The calculation of *DR* brings up a number of interesting questions, including how to treat clients defaulting multiple times or fully repaying their debt in the period under scrutiny etc. These questions will not be discussed in this paper. In general, the percentage of defaulters provides an acceptable approximation.

rate of defaulters in a specific sample over a definite period is described by the Bernoulli distribution. It is also easy to see that the maximum likelihood estimate of the probability of default is equal to the default rate. The problem arises from the fact that most samples used for estimation relate to multiple periods (i.e. the default rate within a cohort in years 2013, 2014 etc.). This entails that *X* changes, too, and therefore the estimator will also be more complex.

As explained above, probability of default *PD* and conditional probability of default *PD*(*X*) are prominent variables in the Vasicek model. Essentially, the model proposes that the general state of the economy can be inferred from the difference of these two variables (their transformed values) or, in other words, there is a long-term centre around which observed probabilities of default fluctuate depending on the general state of the economy.

In this respect, the most remarkable outcome is that within a portfolio of *n* loans of a probability of default of *ρ*, under the assumptions of the Vasicek model, the unconditional rate of default will move toward a Vasicek distribution as *n* increases.

$$F(x) \coloneqq P[X < x] = \int_{-\infty}^{\infty} P[X < x|Y = y]n(y)dy = N\left(\frac{\sqrt{1-\rho}\cdot N^{-1}(x) - B}{\sqrt{\rho}}\right).$$

This means in practical terms that, according to the model, observed default rates follow a Vasicek distribution within a sufficiently large portfolio over a sufficiently long time horizon (*Schonbucher*, 2000).

When variable *X* follows a Vasicek distribution, $N^{-1}(X)$ will have a normal distribution, i.e.

$$N^{-1}(X) = \frac{B_i - \sqrt{\rho}\cdot Y}{\sqrt{1-\rho}} = \frac{N^{-1}(PD_i) - \sqrt{\rho}\cdot Y}{\sqrt{1-\rho}}$$

$$\frac{B_i - N^{-1}(X)\cdot\sqrt{1-\rho}}{\sqrt{\rho}} \sim N(0,1)$$

Consequently,

$$E[N^{-1}(X)] = \frac{N^{-1}(PD_i)}{\sqrt{1-\rho}}$$

$$\sigma^2[N^{-1}(X)] = \frac{\rho}{1-\rho}$$

Based on the foregoing, default rate *DR* follows a Vasicek distribution. As a result, the correct estimator of the unconditional probability of default *PD* in the case of *m* observations is as follows:

$$\widehat{PD}_i = N\left(\frac{\sum_i^m N^{-1}(DR_i)/m}{\sqrt{1+\hat{\sigma}^2}}\right) \text{ és } \hat{\sigma}^2 = \frac{1}{m}\cdot\sum\nolimits_i^m \left(N^{-1}(DR_i)\right)^2 - \left(\sum_i^m N^{-1}(DR_i)/m\right)^2$$

From a statistical point of view, the application of the foregoing is problematic in several respects. For instance, it is not clear how the estimation method should be adapted to small-size portfolios.

Moreover, the prevalent practice among credit institutions is to derive the unconditional probability of default from the average of long time series data (i.e. as a long-term average of default rates for discrete periods). Although this is an efficient estimation, it is not optimal.

Curiously, due to the statistical method applied, unconditional *PD* is often referred to as through-the-cycle *PD* or PD_{TTC}. For the same reason, conditional *PD* is called point-in-time *PD*, denoted by PD_{PIT}. It is most extraordinary to replace a mathematical concept by one referring to the estimation method.

Even if we dispose with economic justification and simply employ a two-parametric distribution for the statistical modelling of conditional probability of default, it must still be assumed that B_i is constant in respect of an asset class over the period under scrutiny. In credit risk jargon, PD_{TTC} must be constant over the estimation period[9]. The legislation and supervisor's recommendations generally address this requirement by putting an emphasis on the TTC/through-the-cycle property. However, no guidance is provided as to the conditions necessary for this variable to be actually constant. When the structure of the economy, the lending practice etc. changes (which are called 'structural changes'), B_i may also change. Accordingly, regulatory authorities legitimately insist that estimations of PD_{TTC} should be based on long time series. In the case of mortgage loans, for Hungary at least, a large proportion of the observations date back to the period of active FX mortgage lending. However, this is a practice that has been virtually – and correctly – banned by the central bank, which can definitely be regarded as a structural break. Similarly, due to the introduction of 'responsible lending rules', collateralized loans have all but vanished in the retail sector. The theory does not provide any clue as to the exact period for which a constant PD_{TTC} may reasonably be assumed.

9 Within the framework of the original model, if B_i is constant, the level of indebtedness of the company is constant. It seems far-fetched to assume that companies' indebtedness is independent of economic cycles. In the first application of the Vasicek model, presented by an analyst company KMV ('V' standing for 'Vasicek' in the acronym), probability of default changed in line with both asset value and indebtedness.

3. PROBLEMS IN THE PRACTICAL APPLICATION OF THE MODEL

There are two important components in the procedure used in practice to calculate the capital requirement.

The estimation method assumes that the default rate is observed on large portfolios that are homogeneous in terms of *PD*. However, when carrying out analyses, it is no known which transaction belongs to which homogenous portfolio. Credit institutions develop a method by which they classify transactions into homogenous groups according to probability of default. Most often a statistical model is used for the definition of the rules of classification, based on behavioural and/or socio-demographic data. Generally, banks group their transactions into rating categories using a score function. The primary purpose of the classification mechanism is the correct categorisation of transactions[10]. Banks estimate the *PD* of each category, mostly independent of the mechanism used for categorisation[11]. Banks can develop their models completely at their own discretion, subject only to general regulatory provisions.

The second component of the procedure is capital requirement calculation. Using the formula laid down in the legislation, which is based on the Vasicek model, credit institutions determine the risk weights of the different assets and based on it, calculate the risk-weighted asset value. At this step, credit institutions have no margin of discretion.

A strange duality is noticeable here. On the one hand, the *PD*s of the rating categories are considered as a given (also called *PD* classes or the 'master scale') and clients are allocated accordingly. For example, the estimated *PD* of category A is below 0.5%. Accordingly, each transaction where behavioural data show a *PD* below 0.5% are classified into this category. On the other hand, the *PD*s of the rating categories are also re-calculated continuously, so the thresholds may change, too. This duality of procedure is not always clear-cut in the practice of different banks and they may even be mixed at times[12].

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10 In accordance with classic Hungarian accounting rules, transactions were assigned to rating categories I to V based on expected loss. In this procedure, *PD* is the only basis of classification. Here lies a major difference as a loan characterised by adequate coverage but a high *PD* would be given a more favourable rating using the logic of Hungarian accounting rules.

11 Consequently, the capital intensity of the respective rating categories (in case of similar average *LGD*s) is fairly constant and the sole purpose of rating is to define the exposure amount for each rating category.

12 That procedure is also called 'calibration' or 'mapping'. The name of the term may originate in the fact that the results of most classifications correspond rather to *PD(X)*, while the probabilities of default of the respective categories are closer to *PD* values. Therefore, *PD(X)* values are mapped to *PD*s. The assumption behind this practice is that the longer the time series used for estimating the *PD*, the better it will approximate the unconditional *PD*.

Most credit institutions employ a rating system in the approval of applications. This process is often based on statistics and as such, involves the determination of a probability of default. Rating systems may have diverse input. Overall experience shows that variables describing the behaviour of borrowers give far more reliable results than financial or socio-demographic indicators. Unfortunately, no database exists at present that would allow any credit institution to monitor the behaviour of borrowers. As a result, new loan applicants are generally rated based on a less efficient rating system. This is less of a problem in corporate lending, where there are few brand new clients. It is, however, typically the case in retail lending (including lending to micro-sized enterprises) in Hungary. At the same time, the models used for the classification of assets rely on the more effective behavioural variables[13]. The final result is that the models used for credit approval are different from those used for capital calculation in the case of most institutions, especially in retail lending.

For creating homogenous portfolios and defining the corresponding *PD*s, credit institutions first determine the development sample and set a target. They are generally the sum of annual periods consisting of overlapping quarters. A score function is established based on this sample, typically using historical behavioural and payment variables (i.e. an aggregated variable – the score – is constructed from several potential explanatory variables). The next step is usually to recalculate the default rate for each score range. Finally, the score ranges (the transactions falling into them) are assigned a *PD* value (or mapped onto a master scale). If the default rate for the examined sample differs significantly from the default rate observed in the underlying sample of the master scale, the *PD* of the master scale may also be adjusted.

This procedure implies an assumption that the average time elapsed since disbursement is fairly constant. However, it is a widely-observed phenomenon, especially in retail lending, that the default rate is not constant over time. For instance, when there is a boom in new transactions, the sample will contain an increased number of more recent transactions, which automatically improves the default rate. This may lead to distortion, particularly in the case of mortgage portfolios.

The distortion caused by overlapping periods is not transparent either.

The relevant legislation defines default in detail and sets the target fairly precisely. However, even in the case of companies, it is very far from Merton's original definition. It is not clear, for example, why should the event of being in default of over

13 Strangely, behavioural models involve some asymmetry, at least in the case of retail loans. A client who was rated high-risk under the behavioural model should be regarded high-risk for the purpose of a new loan as well. However, this is not true at the other end of the risk rating scale. A low-risk rating based on behaviour should not necessarily result in a low-risk rating for the new loan.

90 days correspond to the event that the market value of a company's assets falls below the nominal value of its external liabilities. Or the other way round, what is the precise threshold the market value should fall under when the company's debt is 90 days past due? Of course, it is possible to draw the following formal correspondence: the probability of debt 90 days past due is associated with a threshold B below which A falls with exactly the same probability. However, that would be completely devoid of the economic justification arising from Merton's approach. In this context, during the crisis in Hungary, the supervisory authority estimated that 30% of borrowers who had been defaulting on their mortgage loans for more than 90 days could actually meet their repayment obligations.

It is also problematic that the default rate, in a strict sense, can be established for a period only, so PD^{PIT} cannot be examined. Default rates are typically calculated for at least a quarterly horizon, which hardly meets the point-in-time requirement. This problem is generally eliminated by expert adjustments.

As a long time series is used for estimation, the estimated PD corresponds more to TTC, however, that cannot be taken for granted. For this reason, it is important that these models are used only for allocation to asset classes. However, the PIT and TTC estimates for each class should be established independent of the classification model. These two steps are often not separated in the practice of credit institutions.

4. SUMMARY

There are problems at different levels in the application of the Vasicek model for the quantification of credit risk and the calculation of the capital requirement. Of these, the following two should be highlighted.

1) Both the economic and the statistical model are applicable to corporate portfolios. It is not clear how they should be extended to retail portfolios.
2) The Vasicek model is a one period model. However, a multi-period dynamic model would be more appropriate to the definition of PIT and TTC PD, their relationship and the characteristics of the samples which cover different periods.

The second issue may lead to the overestimation of the probability of default in times of crisis and its underestimation in times of boom. To provide a simple corporate example within the limits of the original model, corporations typically reduce debt during crises (B is lower) and they tend to increase it during booms (B is higher). This means that unconditional PD, i.e. the centre of fluctuation is lower during crises and higher during booms.

The introduction of regulatory provisions and models for the quantification of credit risk definitely had positive effect on banking practices. Nevertheless, their limitations shave also become apparent by now. Yet they have an increasingly important role as the calculation of the capital requirement comes under intense regulatory scrutiny.

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