WHEN IS PREPAYMENT WORTHWHILE?

A theoretical model for evaluating the prepayment option

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ABSTRACT

One of the peculiar perspectives of interest rate risk in the banking book (IRRBB) is the evaluation of so-called embedded options and the quantification of their impact on the value of bank portfolios. One unequivocal characteristic of mortgage portfolios is the option of prepayment, providing the borrower the possibility of redeeming their debt before maturity. There is a broad range of available literature on the evaluation of prepayment options on American markets, but these models cannot be translated to European markets due to structural differences on the latter. In the absence of banking data, I have created a theoretical model for evaluating the prepayment option, based on which it can be demonstrated that, depending on the composition of the given bank portfolio (interest rate level, term to maturity), the prepayment option may have a significant effect on the sum of short-term (i.e. one-year) interest income, as well as on the discount value of the bank portfolio via changing cash flows, and through this the value of economic capital.[1]

JEL codes: G21, G28, G29

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1. A TIMELY TOPIC

In itself, managing interest rate risk is not a new thing for banks, and there are well-developed methodologies for the quantification, coverage and efficient monitoring of risk. The special handling of interest rate risk in the banking book (IRRBB) came to the fore with the creation of the Basel II regulations (BCBS, 2004; EC, 2006), and with the elevation to the regulatory level – under the second pillar of Basel II – of the logic for calculating economic capital. The regulation supplemented the minimum capital requirement defining statutory capital reserves with the second pillar relating to a bank's assessment of its own risks, whereby it is necessary to measure all relevant risks for which capital must be set

1 The author is a PhD student at Kaposvár University. The present article has been written within the framework of her doctoral thesis, and its contents exclusively reflect the opinions of the author.

aside according to the bank's own methodology. IRRBB features among the risks to be quantified under the second pillar. Although the regulations do not specify a mandatory methodology for calculating risks under the second pillar, a number of supervisory recommendations have seen the light of day in support of this.

The special significance of IRRBB is apparent in the fact that, of the risks named in the second pillar, IRRBB is the only one for which the regulator also expects a stress test to be carried out, and a quasi-statutory formation of capital based on the results of such a test (BCBS, 2004; EC, 2006). Regulatory guidelines in recent times also point to the importance of this risk. Spring 2014 saw a proposal emerge for the management of risk under the first pillar, elaborated by the Basel Committee's Task Force on Interest Rate Risk in the Banking Book (TFIR), which did not win the support of finance industry representatives (IIF, 2014). This same proposal was also incorporated into a consultative document published in June 2015 (BCBS, 2015) as one option for a revised IRRBB assessment methodology. The need for close attention to the management of risk is supported by the generally low interest rate environment, and by anxiety that the banking system should set aside adequate reserves to prepare for the risks arising from expected rising interest rates.

IRRBB can essentially be attributed to peculiarities arising from the balance sheet pricing structure: due to their different maturity structures, assets and liabilities have different pricing and repricing properties, and are repriced according to different reference yields that do not correlate perfectly. A further characteristic of balance sheet items can be traced to the behaviour of customers: on the one hand, in the case of liability items without a contractual maturity, depositors may react to changes in the interest rate environment in different ways (deposit movements); while on the other hand, debtors have the option of prepayment of their debts prior to contractual maturity – a decision that is not always made in a financially rational manner. These effects taken together we refer to as the option risk. Balance sheet changes arising from the behaviour of customers cannot be predicted with certainty, and constitute an impact which can be partly traced to financially rational decisions in response to changes in the interest rate environment, and partly to behavioural patterns that can be predicted based on customers' other characteristics.

We can measure the impact of IRRBB in a different way from trading book[2] items. While in the case of trading book items, assets are revalued daily on a

2 The trading book includes financial assets which the bank holds in its own portfolio for the purpose of sale and in order to achieve an exchange rate gain or other price or interest gain. It also includes transactions serving to hedge positions taken up in the trading book, as well as transactions reducing the risk of such positions on the OTC (off-exchange trading) market and active or passive repo transactions.

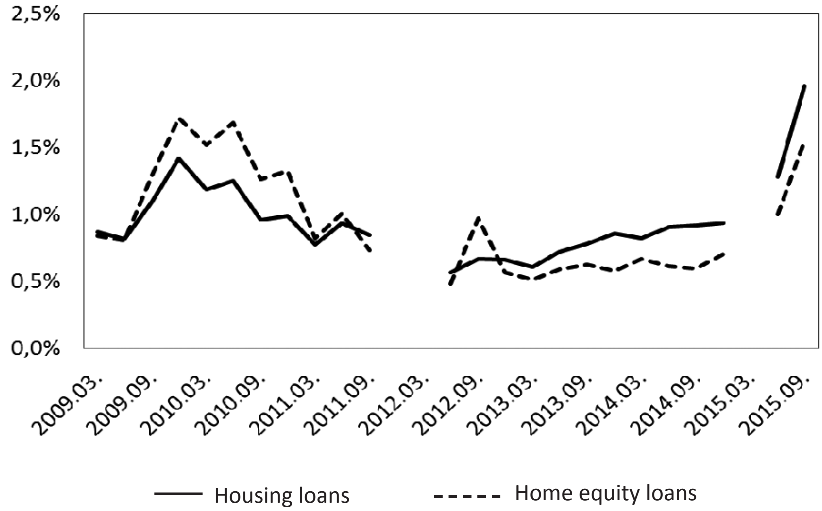
mark-to-market basis and the scale of potential profits/losses arising from interest rate movements can thus also be measured daily, in the case of the banking book we want to measure the effect of changing interest rates with respect to items that have no markets, and thus have no market value to attain. For this reason, no market standards for measuring IRRBB have evolved along the lines of the value-at-risk (VaR) methodology for market risks.

The other reason that evolved methods for measuring market risk cannot be applied without modification is the existence of numerous assumptions pertaining to items in the banking book, which have an impact on the process of quantifying risk. This surfaces primarily because a substantial number of banking book items conceal so-called embedded options, which make it difficult to model the expected cash flows from these assets. Typical examples of this are demand deposits and prepayment options pertaining to mortgage loans. Option features affect cash flows from banking book items, and consequently also impact risk exposures and the bank's level of economic capital, so that – depending on what assumptions we make in determining the risk exposure from these items – they can significantly influence the level of economic capital. We have only partial knowledge of the impact of these option features on economic capital, and banking practices are diverse in their approach to quantifying these; moreover, the available literature also recommends a variety of approaches, select aspects of which I will present in the following.

The proportion of prepayments on mortgage loans in Hungary amounts to 1% on average going back to 2009, not taking into account the effect of preferential final payments not induced by market processes. A shifting trend can be observed in the scale of prepayment following settlement of FX-denominated loans.

Figure 1

Proportion of prepayments of mortgage loans over time

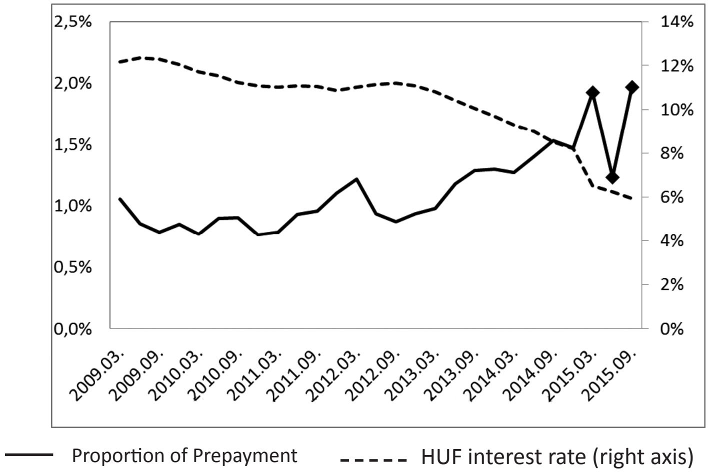


Source: National Bank of Hungary (MNB), own calculations, own design[3]

A significant discrepancy is perceptible in the scale of prepayments in the case of forint or foreign currency-based credit portfolios: while with forint portfolios, the proportion was stable at around 1% on average, in the case of foreign currency-based loans the rate sunk from the level prior to final payment to the 0.5% level, where it stabilised. Following foreign currency settlement, the level of prepayment began to rise, supported by both regulatory relaxation and the falling forint interest rate. The market climate favours prepayment, lending an added timeliness to the measurement and modelling of prepayments.

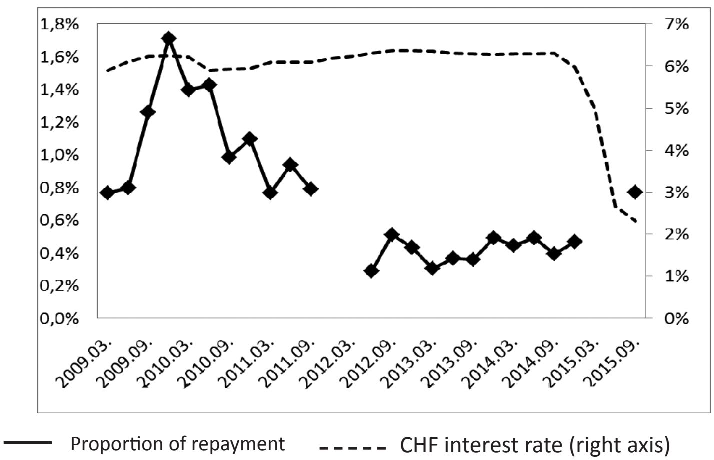
3 The effects of preferential final payments and settlement of FX-denominated loans do not feature in the diagram, since the one-off impact of these distorts the outlying trends.

Figure 2
Proportion of prepayments of forint-denominated home mortgage loans and average interest rate levels over time



Source: National Bank of Hungary (MNB), own calculations, own design[4]

Figure 3
Proportion of prepayments of Swiss franc-denominated home mortgage loans and average interest rate levels over time



Source: National Bank of Hungary (MNB), own calculations, own design[5]

4 The rate of prepayment of portfolios converted into forints following settlement of FX-denominated loans is shown by the data from 2015.

5 The effects of preferential final payments and settlement of FX-denominated loans do not feature in the diagram, since the one-off impact of these distorts the outlying trends.

2. ECONOMIC CAPITAL MODELS RELATING TO IRRBB

2.1. General economic capital models

The impact of interest rate risk in the banking book can be quantified using two approaches, based on the available literature, international supervisory recommendations and the practice of major international banks. The *income-based approach* measures the effect of interest rate movements on bank income, primarily on net interest income (NII), while the *economic capital-based approach* aims to quantify the degree of change occurring in the (present) value of capital based on reevaluation of the bank's future portfolio cash flows.

Van Mullem (2004, p. 39) cites *Bessis* (1998) and *Matten* (1996) as stating that "either an earnings approach or a value approach can be taken to calculate the economic capital requirement. In fact, the income approach equals the earnings-at-risk technique, whereas the value approach equals the duration (*Bessis*) and value-at-risk (*Matten*) techniques." According to these approaches, in the income-based methodology the volatility of interest rates (*Bessis*, 1998, in: *Van Mullem*, 2004, p. 39) or the volatility of earnings (*Matten*, 1996, in: *Van Mullem*, 2004, p. 40) is the source of risk, while the largely equivalent value-based methodology is based on the volatility of the net present value of the banking book (*Van Mullem*, 2004, p. 40).

Likewise cited by *Van Mullem*, the methodology applied by Oliver, Wyman & Company (2001) "defines interest rate risk as the volatility of a book's net asset value or the present value of equity" (*Van Mullem*, 2004, p. 41). The OWC model also incorporates measurement of the effect of customer behaviour via the impact of changes in the interest rate environment. Another important part of the concept is to take into account possible management intervention, which is the assumption that "management will intervene if losses were to become too big. Thus, the actual loss will be less than the maximum calculated loss, depending on the point of time the management intervenes" (*Van Mullem*, 2004, p. 43).

The model of *Emmen* (2001) tries to determine the "maximum amount of value that can be lost in a year due to unexpected rate movements." In this approach, "economic capital is the difference between today's market value and the worst-case market value after one year. The market value after one year is the present value of the position at that time plus the interest income in the coming year" (*Van Mullem*, 2004, p. 44).

There are advantages and drawbacks to all the aforementioned economic capital models. I summarise the structure of these capital models in *Table 1*.

Table 1
Summary of selected economic capital models

	Bessis & Matten	OWC	Emmen
Income-based	yes	no	no
Capital value-based	yes	yes	yes
Definition of interest rate risk	volatility of banking book NPV	volatility of banking book NPV	change in market value of balance sheet
Balance sheet structure assumption	stable	stable, going concern	dynamic
Customer behaviour taken into account	no	yes	yes
Covered risk factors	repricing yield curve optionality (partial)	repricing yield curve optionality (partial)	repricing yield curve

Source: own design, based on *Van Mullem* (2004)

2.2. Potential models for the prepayment option

In specialist writing on the topic, a fundamental distinction is drawn between financial models that take the optimal decision-making mechanism as their basis, and models that also quantify the impact of factors deflecting individual decision-making from the optimal. With respect to modelling techniques, significant differences are apparent in the characteristics that define US or European markets. American markets are characterised by the securitisation of primary claims on secondary markets, where forecasts of the expected cash flows of such securities are necessary for their pricing; here, it is primarily prepayments of loans that may alter the originally planned cash flows, helped by a considerably greater degree of freedom in prepayment. The structure of European markets differs from this in that prepayment opportunities are generally limited (burdened by substantial costs), and because quantification of the value of the prepayment option and its projected impact on a bank's portfolio is important in terms of its liquidity and capital management, so that scorecard-type modelling approaches built on internal data are consequently also assigned a role.

The many models to be found in the international literature are split by Vasconcelos (2010) into two major groups. One such major group of models takes as its basis "the assumption that prepayment is exercised in an optimal way (where) the mortgagor would prepay when the value of the mortgage is greater than the outstanding debt plus transaction costs" (*Vasconcelos*, 2010, p. 5). The other group of models, on the other hand, assumes "an exogenous prepayment rule (where) actual prepayments often appear to be non-optimal or irrational from a risk-value perspective. In other words, mortgagors may prepay when the prevailing mortgage rate is above their contractual rate" (*Vasconcelos*, 2010, p. 5). This eventuality is not covered by models based on the assumption of optimal prepayment.

A broad range of literature exists that deals with the modelling of prepayment options as it relates to retail mortgage portfolios on the US market. The explanation for this is that the market in mortgage-backed securities (MBS) evolved on the American capital market, where modelling of the prepayment option is a prerequisite for the pricing of these instruments, its impact needing to be taken into account in the cash flow of such securities (*Kalotay* et al., 2004). There is a well-developed secondary MBS market, and consequently the pricing of these instruments occurs on a fundamental market basis. This market structure does not apply to European markets, however, where the financing model is typical. For this reason, American models are not applicable to European mortgage markets in unchanged form.

One essential American empirical model is the study by *Kang* and *Zenios*, also often referred to as the *Wharton prepayment model* (*Kang–Zenios*, 1992). The variables it contains also determine the later framework of empirical studies that apply to other markets. The Wharton model contains four variables as explanatory prepayment variables (*De Vreede*, 2008, pp. 23–24):

- *Refinancing incentive*: a variable measuring the absolute or relative difference between the prevailing mortgage yield R and the coupon rate C of the given mortgage loan. The indicator expresses the financial incentive for prepayment.
- *Seasonality*: a variable expressing the seasonal factor observable in prepayment rates. Observations show that the level of prepayment is generally higher in summer and lower in winter.
- *Seasoning*: a variable indicating that prepayment is lower in the first years of a mortgage loan's lifetime, while the rate rises continuously thereafter.
- *Burnout*: a variable expressing the maturity effect of seasoning, which can be interpreted at the portfolio level. Burnout describes the effect whereby prepayment decreases as the mortgage portfolio ages, i.e. as maturity

approaches. This may be attributable to several factors. In the case of a mortgage portfolio, as soon as a positive incentive exists for prepayment, then the most active mortgagors will probably be the first to take advantage of the opportunity, while others will either wait for further interest reductions or, with the change in the interest rate environment proving neutral for them and unable to obtain loans elsewhere, will not take advantage of the prepayment option. As soon as a positive prepayment incentive arises once again at a later date, the remaining mortgagors in the portfolio will be less active in their reactions, so that the risk of prepayment decreases, and thus the rate of prepayment across the entire portfolio falls as maturity approaches.

A number of available empirical studies also deal with quantifying the effect of prepayment. In a 2001 study, *Doff* examined the prepayment behaviour of Dutch mortgagors, carrying out an analysis of Rabobank data in the period from 1997 to 2000. *Doff* applied so-called survival analysis to three types of mortgage: annuity, unit-linked and interest-only mortgages. The final explanatory variables in the models were the refinancing incentive, seasonality and seasoning (*Van Mullem*, 2004, p. 143). *Charlier* and *Van Bussel*'s 2001 study established separate models for unit-linked and interest-only mortgages. Regarding unit-linked mortgages, they found that the prepayment rate increases with the age of the mortgage contract. If the burnout factor is excluded from the model, a positive correlation can be observed between prepayment and the refinancing incentive. However, if the burnout factor is included, the effect of the refinancing incentive disappears and its role is taken over by the burnout factor. The type of property likewise proved a strong explanatory variable (*Van Mullem*, 2004, p. 145; *Charlier–Bussel*, 2001, p. 23). In a 2002 study, *Alink* created both a general model and separate models for individual product types, using logistic regression. He developed models based on data from the Dutch SNS Bank and back-tested them on data from DBV and Rabobank. Explanatory variables included in his final model were seasoning, refinancing incentive, loan-to-foreclosure-value ratio, age of the borrower, interest rate movement, market rate and rank of the mortgage. Further dummy variables expressed whether the mortgage was sold through an intermediary, the property type, mortgage type, and which interest rate-fixing period the mortgage had reached (*Van Mullem*, 2004, p. 146). In 2008, *De Vreede* examined prepayment behaviour in the mortgage portfolio of Fortis Bank Mijdrecht, a branch of Fortis Bank. The analysis distinguished between exogenous and endogenous variables that may influence prepayment. The model building led to the selection of the following relevant variables as explanatory variables (*De Vreede*, 2008):

- *Exogenous variables*: refinancing incentive, steepness of the yield curve, direction of interest rate changes, level of interest rates.
- *Endogenous variables*: seasoning, loan-to-foreclosure-value ratio, mortgage rank, age of the borrower, property type, property's geographic location, interest type, mortgage type, distribution channel, distance from bank branch.

All the above studies demonstrate that prepayment has a significant impact, as each revealed a substantial rate of prepayment.

With respect to the UK market, an analysis appeared in 2001 taking an actuarial approach to the experience of modelling mortgage prepayment. In their study, the authors distinguish two different modelling possibilities (*Perry* at al., 2001, p. 4):

- *Models that assume optimal prepayment*: these models are applicable for the modelling of the prepayment option, and thus for predicting the NPV effect in the banking book, when prepayment is a financially rational move for mortgagors. These models cover the majority of prepayment eventualities.
- *Models that assume sub-optimal prepayment*: these models take into account eventualities when prepayment is not financially rational and is triggered by other circumstances, e.g. prepayment deriving from substantial savings, sale of collateral, etc. These are known as behavioural factors.

The study was prepared with the participation of market players accounting for 65% of the UK mortgage market. Based on these data, the following risk factors were shown to be the most relevant (*Perry* at al., 2001, p. 10):

- *Age of the loan*: having already gone through the process of taking out a mortgage loan, most borrowers do not want to refinance or move home again, and consequently prepayment is more typical in the second half of the lifetime of the loan.
- *Property price changes*: when house price inflation is high, the number of transactions on the property market increases, resulting in a higher rate of prepayment.
- *Interest changes and interest rate differential*: the interest rate differential is the difference between the current mortgage interest and a rate offered by another lender; observations show that the higher the interest rate differential, the higher the prepayment activity.
- *Prepayment charges*: prepayment charges above a certain cost level reduce the inclination to prepay.

Within the framework of UniCredit & Universities, a study by *Consalvi* and *Scotto di Freca* was published in 2010 on the results of prepayment modelling as it applies to UniCredit's mortgage portfolio. Using the survival analysis methodology, the authors established behaviour-based scorecards for fixed-rate and variable-rate mortgage portfolios. In their study they describe how measurement of the prepayment option can be attributed to two theoretical approaches: the financial approach, which is based on the principle of arbitrage and which they trace back to option evaluation models; and the behavioural approach, which they map with econometric models. The financial approach is suitable for the evaluation of callable securities, while the behavioural approach is appropriate for analysing prepayment trends in retail mortgage portfolios. Within the behavioural approach, a prominent role is assigned to so-called survival models, on which the authors also rely in their own study (*Consalvi–Scotto di Freca*, 2010, pp. 2–3). The authors developed their models for the portfolio of UniCredit retail mortgages prepaid in the period 2005–2009. Separate models were established for fixed-rate and variable-rate mortgage loans. The most important distinction between the two model types was that in the case of fixed-rate loans the refinancing incentive was also included in the model as a relevant variable. With this exception, both models incorporated almost the same set of variables among the data characterising borrowers: namely, the age of the borrower, their nationality, occupation and the sector in which they work, and the original maturity date of the loan (*Consalvi–Scotto di Freca*, 2010).

In the following chapter I present a general theoretical framework built on elements of a simplified economic capital model, and taking into account optimal prepayment.

3. A THEORETICAL MODEL

3.1. General framework

Below I make an attempt to model the impact of the prepayment option on the value of bank portfolios through a general example. Given that there are no actual bank data available for calculations, the modelling focuses on the possibility of modelling the optimal prepayment option and quantifying its theoretical impact. It follows from this that I do not venture into an examination of the impact of exercising prepayment options based on individual, non-optimal decisions.

I carry out the modelling on a hypothetical bank portfolio, presenting the general logical framework of the model through a simple example. I take a loan portfolio containing four elements, its parameters contained in *Table 2*. The interest income cash flows on the loan portfolio and the current yield curve evolve as seen in *Table 3*.

Table 2
Elements of a hypothetical loan portfolio

	Loan 1	Loan 2	Loan 3	Loan 4
Amount of loan	1 000 000	1 000 000	1 000 000	1 000 000
Coupon rate	5%	6%	7%	8%
Remaining term to maturity (years)	5	6	7	4

The calling in of the prepayment option is presaged by establishing the refinancing incentive. This approach is an accepted methodology in the specialist literature, and the majority of prepayment models apply it according to various definitions. The refinancing incentive is determined by the par yield curve related to the current spot yield curve. Inasmuch as the par interest rate[6] applying to the given remaining term to maturity is more favourable than the current coupon rate, then prepayment of the loan occurs on the assumption that an optimal decision is being made.

Table 3
Interest income cash flows of loan portfolio and yield curve

Year	CF1	CF2	CF3	CF4	r
1	50 000	60 000	70 000	80 000	6.0%
2	50 000	60 000	70 000	80 000	5.8%
3	50 000	60 000	70 000	80 000	5.6%
4	50 000	60 000	70 000	80 000	5.4%
5	50 000	60 000	70 000		5.2%
6		60 000	70 000		5.0%
7			70 000		4.8%

Source: own design, based on own calculations

I carry out the examination for two points in time: along the current yield curve, and along the yield curve one year later, assuming the situation at that time (it will be substituted by the modelling of the yield). The reasoning for examining these two points in time is that the planning cycle is generally a year long, and for this reason it is worth narrowing down examination of the impact on interest income to this time band. A further assumption is that as soon as the loan is

6 The par interest rate is the nominal interest rate (coupon rate) at which a bond can be issued at nominal value along the current spot yield curve.

prepaid, the prepaid principal is refinanced at the new par interest rate for the remaining term to maturity, thus altering the cash flow on the loan portfolio.

From all this, it follows that I measure the impact on interest income by determining the difference between interest income on the original cash flow and interest income on the new cash flow after prepayments. Staying with the example, I illustrate the calculations in *Table 4*.

Table 4
Expected prepayments on the loan portfolio

Year	CF1	CF2	CF3	CF4	r0	par0	r1	par1
k	*5%*	*6%*	*7%*	*8%*				
1	50 000	***60 000***	***70 000***	***80 000***	6.0%	6.00%		
2	***50 000***	60 000	70 000	80 000	5.8%	5.81%	5.8%	5.80%
3	50 000	60 000	70 000	80 000	5.6%	5.61%	5.4%	5.41%
4	50 000	60 000	70 000	80 000	5.4%	***5.43%***	5.0%	5.03%
5	50 000	60 000	70 000		5.2%	5.24%	4.6%	***4.64%***
6		60 000	70 000		5.0%	***5.06%***	4.2%	4.27%
7			70 000		4.8%	***4.88%***	3.8%	3.89%

Source: own design, based on own calculations

Along the current yield curve (*r*0), Loans 2–4 are worth prepaying in the first year, since the par interest rates along the par yield curve for these loans' remaining term to maturity are more favourable, meaning that borrowers can have these loans refinanced at a lower coupon rate. In the case of Loan 1, this is a realistic option only along the yield curve one year later, and consequently prepayment of this loan occurs one year later. Assuming that the repaid loan amount is refinanced at the new par interest rate, the cash flow on the bank's loan portfolio changes, the results being shown in *Table 5*.

Table 5
New interest income cash flows of loan portfolio

Year	CF1	CF2	CF3	CF4
k	*4.64%*	*5.06%*	*4.88%*	*5.43%*
1	50 000	50 569	48 751	54 263
2	46 449	50 569	48 751	54 263
3	46 449	50 569	48 751	54 263
4	46 449	50 569	48 751	54 263
5	46 449	50 569	48 751	
6		50 569	48 751	
7			48 751	

Source: own design, based on own calculations

The impact of the prepayment option on the interest income of the hypothetical loan portfolio, projected on a one-year phase of the forecast period, can be expected to result in a 22.7% decrease. The detailed results can be seen in *Table 6*.

Table 6
Impact of prepayment on interest income cash flows of loan portfolio

	Cash flow
Original interest income	1 420 000
Altered interest income	1 097 518
Change	–322 482
% change	–22.7%

Source: own design, based on own calculations

If we wish to quantify the impact on the present value of the banking book, cash flows must be supplemented with repayments of principal, and the change in the present value of the bonds thus obtained must be determined. The results are shown in *Table 7*.

Table 7
Impact of prepayment on present value of loan portfolio

	Present value
Original cash flow	4 248 982
Modified interest income	3 977 905
Difference	–271 078
% change	–6.4%

Source: own design, based on own calculations

In carrying out calculations, for the sake of simplicity I make the assumption that only interest is paid in individual periods, with repayment of the principal due in a lump sum at maturity. In reality, retail mortgage loans are repaid in annuities. Essentially the simplification of assuming repayment of the principal was necessary so that I could apply a suitably simple calculation method by programming the calculations into the Excel VBA program. Annuity repayment would refine the calculations somewhat, but without affecting the final result and conclusions. In the case of annuity and bullet cash flows (the latter entailing repayment in one sum at the end), the difference in the cash flow impact increases as the refinancing interest level decreases. The correlation is linear. On this basis, the conclusion can be drawn that the results of annuity and bullet-type calculations, based on linear correlations, can be mutually reconciled. In the case of bullet-type loans, a consistently higher cash flow effect is also evident, meaning that with this method the end result is overestimated. Based on the above, it can be stated that the simplification applied to the calculation method does not distort the end results, and that it is appropriate for conclusions to be drawn.

For the modelling of the yield curve, I use the Cox-Ingersoll-Ross model (CIR), while applying theoretical parameter settings in the course of calculations. The reason for doing this is in order to examine how sensitively the end results react to changes in these settings. CIR is an equilibrium yield curve model in which interest rates cannot be negative. According to the model, the process of evolution of the instantaneous interest rate (r) in a risk-neutral world is:

$$dr = a(b-r)dt + \sigma\sqrt{r}dz,$$

where a, b and σ are constant. Mean reversion prevails in the model, with deviation proportional to $\sqrt{r}$. This means that if the short rate increases, then so too does its deviation.

3.2. The hypothetical bank portfolio and the calculation process

When constructing the model, I needed to include some significant simplifications with respect to the composition of the examined loan portfolio. For the sake of simplicity, the loan portfolio comprises five elements, each representing a sub-portfolio. These sub-portfolios vary in average interest rate levels and remaining term to maturity, and I summarise their characteristics in *Table 8*. The aim of the portfolio settings is to obtain diversity in both interest levels and maturities, similarly to the composition of real-life portfolios.

Table 8
Composition of the hypothetical loan portfolio

Sub-portfolios	1.	2.	3.	4.	5.
Weight in the portfolio	20%	20%	20%	20%	20%
Average interest rate level	4%	5%	6%	7%	8%
Average remaining term to maturity (years)	10	5	6	7	4

For the interest rate environment, I make initial assumptions of a short rate of 6%, returning to the 4% level in the long term. I apply the CIR model using the parameters shown in *Table 9*. The set interest rate level is adjusted to the interest rate composition of the loan portfolio.

Table 9
Parameters of the CIR model – declining yield curve

	CIR parameters
r_0	6%
a	0.5
B	4%
ρ	5%

The logical framework of the model can be summarised as follows:

1. *Yield curve modelling*. Based on the CIR model, I model the short interest rate and related points on the yield curve for a 30-year time horizon, at monthly intervals. The possible trajectories of the short rate are modelled at monthly intervals ($\Delta t = 1/12$) for the remaining term to maturity of the individual loan portfolio elements.
2. *Determining the par yield curve*. For every individual yield curve I establish par yield curves. I use par yield curves in the approach to current refinancing interest rates, assuming that the loans are priced fairly and refinancing can be obtained at the par interest rate on the market. To simplify the calculations, I do not adjust the par interest rates for the individual risk of the loan portfolio, since essentially this only displaces the interest levels and does not influence the refinancing decision-making mechanism. In this way the model can easily be supplemented.
3. *Determining the refinancing incentive*. I determine the refinancing incentive based on a comparison between the par interest rate for the given remaining term to maturity and the average interest level of the loan portfolio. I con-

tinue the comparison right up until the point the simulated par interest rate drops below the coupon value, but until the end of the term to maturity at the latest. If the simulated par interest rate falls below the coupon value, and assuming the principle of optimal prepayment, then prepayment occurs. I carry out the calculations without taking into account prepayment charges, presuming that prepayment can be made without restriction, as well as by also taking these charges into account. By taking the prepayment charges into account, it is possible to analyse the extent to which these charges may influence the possibility of optimal prepayment.

4. *Determining the interest income effect.* If the par interest rate falls below the coupon value, and prepayment occurs as a consequence, then I make the assumption that the repaid principal is refinanced at the current interest rate, i.e. at the par interest rate. Cash flow for the remaining term to maturity, or the difference between the original cash flow and the altered cash flow, is calculated based on the new interest rate. I determine the cash flow effect both without a discount and based on discounted cash flow. The cash flow effect is useful for examining the impact of the income-based approach, the goal of which is to estimate the interest income effect. The goal of the discounted cash flow effect is to estimate the change in asset value, and to calculate the impact of the economic capital-based approach accordingly.
5. *Determining results in the case of a stressed interest trajectory.* I repeat the calculations for a stressed interest rate environment for two reasons: 1. the above interest environment modelling assumes normality, which is adequate in the case of the normal course of business, but unsuitable for simulating potential losses in a crisis situation; and 2. during analysis of the interest rate risk, it is a regulatory requirement to determine the impact on economic capital value under a stressed interest rate environment. For the calculations, I determine the stressed interest trajectory with a parallel shift in the yield curve, the scale of which I calculate based on the 99% one-year VaR value of short forint yields.

In the absence of genuine bank data, the model does not include an examination of the impact of non-optimal decisions. The impact of decisions that deviate from the optimal distorts the impact of optimal decisions, potentially either reinforcing or weakening the latter for a variety of reasons. Non-optimal decisions may be assessed based on genuine bank data, and for this reason I do not take account of the supposed impact of these in the model since their inclusion would entail too many assumptions, calling into question the interpretation of results.

This assumption, in my view, does not weaken the applicability of the model. In a structured analysis, I see it as a step forward if a “chemically pure” situation is the subject of analysis since we do not yet see its possible impact on capital precisely, while the result of such an analysis can represent a starting point for analysis of the scale of further "distorting" effects. Consequently I construct the modelling process according to this logic, first analysing a purely optimal decision-making situation, then modifying this to include the cost factor, and then moving on to include various individual distorting factors. I believe this is possible in the event that the preceding two steps produce a result showing that the impact of option risks on capital alone can be significant, and that consequently this is the range of issues that needs to be addressed. If a more complex behavioural structure were already to be modelled as a first step (which, in the absence of genuine data, could be done only by making some strong assumptions), then the model would not permit us the opportunity to separately analyse the impact of individual elements (optimal and non-optimal decision-making situations).

From a modelling point of view, there are two approaches to the inclusion of behavioural factors. One approach seeks to answer the question of what socio-demographic and other factors explain prepayment. A prepayment behavioural scorecard can be constructed on this basis, enabling the bank to identify and evaluate portfolios that are more exposed to the prepayment risk. A scorecard cannot be developed on the basis of assumptions, and certainly requires elementary internal bank data. The other possible approach is to isolate a number of occurrences that we regard as non-optimal prepayment (e.g. prepayment from an inheritance, from the sale of real estate, etc), and to estimate the scale of these, taking this into additional account over and above the optimal prepayment. The extent of these can be estimated based on historical data, and from the modelling point of view is measured as a percentage value that can be projected onto the portfolio as a whole.

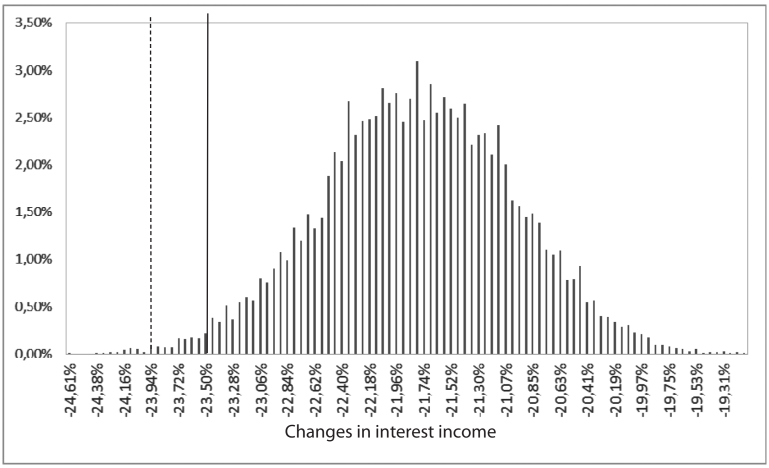
3.3 The income-based approach

3.3.1 Without prepayment charges

Adopting the above logic for hypothetical loan portfolios, I determined the potential impact on interest income using 10,000 simulations. Given that I only examined prepayment when making my calculations, without any new additional loans, only the so-called downside risk – i.e. the negative effect on interest income – was taken into account. Accordingly, the results for the individual sub-portfolios reveal the potential scale of the shortfall in interest income compared to the originally planned interest income amounts for the entire duration of the loans. The calculations examined the effect on cash flow, without any discount effect. I determined the impact on interest income with the assumption that, in the case of prepayment, the prepaid principal is refinanced at the lower interest rate for the remaining term to maturity. In this way, the impact on interest income is the difference between the original interest income cash flow and the nominal value of the changed interest income cash flow.

The distribution of the combined interest income effect across the individual loan portfolio elements is shown in *Figure 4*.

Figure 4
Distribution of interest income effect,
determined based on change in cash flow across entire loan portfolio*



Note: *broken line 99% confidence level, solid line 95% confidence level
Source: own design, based on own calculations

The key statistics relating to the effect on individual loan portfolio elements and the loan portfolio as a whole are contained in Table 10.

Table 10
Statistics of interest income effect – declining yield curve

Sub-portfolios	**1.**	**2.**	**3.**	**4.**	**5.**	**Overall effect**
Coupon rate	*4%*	*5%*	*6%*	*7%*	*8%*	
Remaining term to maturity	*10*	*5*	*6*	*7*	*4*	
Average	–0.87%	–5.61%	–22.84%	–34.90%	–39.44%	–21.81%
Spread	0.77%	2.60%	1.96%	1.51%	1.99%	0.78%
95% confidence level	–2.40%	–9.97%	–26.05%	–37.37%	–42.67%	–23.10%
99% confidence level	–3.36%	–11.82%	–27.39%	–38.36%	–44.08%	–23.62%

Source: own design, based on own calculations

The interest levels of the examined sub-portfolios and the prevailing interest rate environment, as well as assumptions of changes in the latter, have a significant influence on the results. As a consequence of the assumed effect of declining interest rates, the prepayment option had a significant effect in the case of sub-portfolios 3–5, while close to a quarter of the interest income on the portfolio as a whole was endangered. With these sub-portfolios the effects were concentrated in the first 12 months, so that the interest income effect was substantial within the first year.

If we examine the interest income effect for only the first 12 months, meaning that we compare the potential scale of the shortfall in interest income in the first year along the individual interest trajectories with the expected interest income levels in the first year, then the statistics change as shown in *Table 11*.

Table 11
Statistics of interest income effect – declining yield curve, effect within one year

Sub-portfolios	1.	2.	3.	4.	5.	Overall effect
Coupon rate	*4%*	*5%*	*6%*	*7%*	*8%*	
Remaining term to maturity	*10*	*5*	*6*	*7*	*4*	
Average	–2.66%	–21.31%	–35.71%	–45.70%	–49.50%	–34.91%
Spread	6.14%	2.26%	1.58%	1.24%	1.63%	1.04%
95% confidence level	–16.85%	–24.97%	–38.28%	–47.70%	–52.10%	–37.02%
99% confidence level	–17.25%	–26.61%	–39.28%	–48.44%	–53.09%	–37.77%

Source: own design, based on own calculations

The effect within one year appears much more forcefully. The result obtained here is a potential maximum, since I assumed an optimal decision–making mechanism, while not reckoning with prepayment and transaction charges. Accordingly, assuming a declining interest rate environment for the hypothetical portfolio, at a 95% confidence level one third of the planned one–year interest income is potentially endangered. The interest income effect is considerably smaller than this, since the declining interest rates are also apparent in declining funding costs, so that the net effect must be considerably more favourable than the theoretical maximum determined for interest income.

Altered assumptions of the interest rate environment significantly impact results. I also carried out calculations assuming rising interest rates, using the parameters shown in *Table 12*.

Table 12
Parameters of the CIR model – ascending yield curve

	CIR parameters
r_0	5%
a	0.5
B	7%
ρ	5%

With these settings, the key statistics relating to the effect on individual loan portfolio elements and the loan portfolio as a whole are contained in Table 13. In the case of a rising interest rate environment, the impact of the prepayment option on interest income levels is much weaker.

Table 13
Statistics of interest income effect – ascending yield curve

Sub-portfolios	1.	2.	3.	4.	5.	Overall effect
Coupon rate	*4%*	*5%*	*6%*	*7%*	*8%*	
Remaining term to maturity	*10*	*5*	*6*	*7*	*4*	
Average	0.00%	–0.06%	–0.52%	–8.66%	–23.47%	–6.57%
Spread	0.03%	0.23%	0.70%	1.38%	1.84%	0.51%
95% confidence level	0.00%	–0.43%	–1.90%	–10.96%	–26.34%	–7.46%
99% confidence level	–0.01%	–1.12%	–3.20%	–11.78%	–27.93%	–7.83%

Source: own design, based on own calculations

If we examine the interest income effect for the first 12 months, meaning that we compare the potential scale of the shortfall in interest income in the first year along the individual interest trajectories with the expected interest income levels in the first year, then the statistics change as shown in Table 14.

Table 14
Statistics of interest income effect –
ascending yield curve, effect within one year

Sub-portfolios	1.	2.	3.	4.	5.	Overall effect
Coupon rate	*4%*	*5%*	*6%*	*7%*	*8%*	
Remaining term to maturity	*10*	*5*	*6*	*7*	*4*	
Average	0.00%	0.00%	–2.89%	–23.92%	–36.08%	–15.78%
Spread	0.00%	0.00%	6.37%	1.22%	1.47%	1.35%
95% confidence level	0.00%	0.00%	–17.01%	–26.04%	–38.53%	–18.85%
99% confidence level	0.00%	0.00%	–17.51%	–26.73%	–39.44%	–19.28%

Source: own design, based on own calculations

In the case of a rising interest rate trajectory, the potentially endangered interest income – assuming a 95% confidence level – is close to one fifth of annual interest income, meaning that the effect remains significant even when assumptions of the interest rate environment theoretically do not favour prepayment. The scale and nature of the effect are fundamentally influenced by the composition of the examined loan portfolio, since the effect appears in the case of sub–portfolios with a high coupon rate, where – as interest rates start from a low level compared to the coupon rate, and thus rising interest rates can be assumed – it makes sense to prepay. Naturally the result thus obtained can also be regarded as a potential maximum.

3.3.2 With prepayment charges

I also carried out calculations incorporating prepayment charges. For the prepayment charge I made the assumption that a fixed charge of 2% must be paid in the event of prepayment. The prepayment charge has an effect on cash flow through the refinancing incentive. Accordingly, refinancing occurred in the model if the par interest rate for the given remaining term to maturity and the annualized total value of prepayment charges distributed over the remaining term to maturity taken together were lower than the coupon rate. In certain cases, inclusion of the prepayment charge will redirect a refinancing decision based only on the par interest rate, since by taking the charge into account refinancing is no longer worthwhile. I summarise the results of the model calculated by taking the charges into account in *Table 15*.

Table 15
Statistics of interest income effect – declining yield curve, with prepayment charges

Sub-portfolios	1.	2.	3.	4.	5.	Overall effect
Coupon rate	*4%*	*5%*	*6%*	*7%*	*8%*	
Remaining term to maturity	*10*	*5*	*6*	*7*	*4*	
Average	–3.32%	–8.93%	–22.86%	–34.84%	–39.37%	–22.78%
Spread	2.88%	2.75%	1.90%	1.50%	2.00%	0.99%
95% confidence level	–6.55%	–12.27%	–25.94%	–37.23%	–42.65%	–24.25%
99% confidence level	–7.73%	–13.79%	–27.14%	–38.14%	–44.03%	–24.67%

Source: own design, based on own calculations

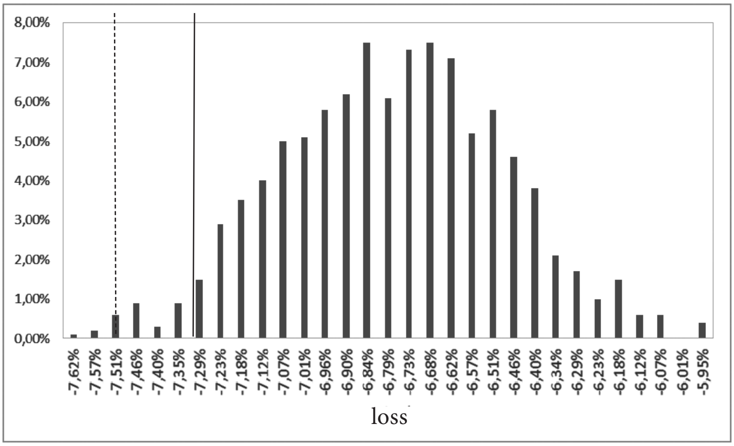
The introduction of the prepayment charge further worsens the interest income effect (comparing the effect to interest income attainable over the entire loan duration). The reason for this is that although prepayment occurs less frequently due to the charge, when the loan is refinanced according to the model this occurs at a lower interest rate on average than when the model contains no prepayment charge.

3.4 Capital value-based approach

In the case of the capital value-based approach, the aim is to determine the change occurring in the value of economic capital due to changing interest rates. For the calculations, changes in the value of assets and liabilities should also be determined, the difference between these providing the change in the value of economic capital and its distribution. During the simulation, I examined the change in the discounted cash flow of the loan portfolio, without taking into account any simulation of the liabilities side, so that the change in the value of the loan portfolio – *ceteris paribus* – results in the change in the value of economic capital. I determined the change in the value of economic capital as a ratio of the changes in the discounted cash flow and the original capital value.

I carried out calculations assuming both a declining and increasing interest rate environment. In the case of a declining interest rate environment, the economic capital effect projected onto the entire loan portfolio is shown in *Figure 5*.

Figure 5
Distribution of economic capital effect, determined based on change in discounted cash flow across entire loan portfolio*



Note: *broken line 99% confidence level, solid line 95% confidence level)
Source: own design, based on own calculations

The key statistics relating to the effect on individual loan portfolio elements and the loan portfolio as a whole are contained in Table 16. In the case of the discounted cash flow effect, the results are lower than in the case of the interest income effect, which is due to the existence of the discount. If we wish to translate the results into the capital requirement, then the results are fit for this purpose.

Table 16
Statistics of economic capital effect – declining yield curve

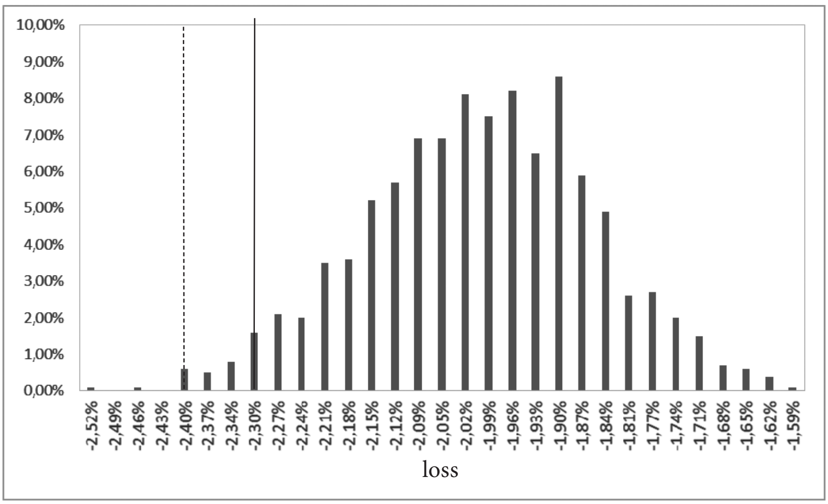
Sub-portfolios	1.	2.	3.	4.	5.	Overall effect
Coupon rate	*4%*	*5%*	*6%*	*7%*	*8%*	
Remaining term to maturity	*10*	*5*	*6*	*7*	*4*	
Average	–0.27%	–1.23%	–7.03%	–14.30%	–11.18%	–6.80%
Spread	0.24%	0.55%	0.61%	0.68%	0.57%	0.30%
95% confidence level	–0.75%	–2.15%	–8.04%	–15.42%	–12.12%	–7.28%
99% confidence level	–1.00%	–2.49%	–8.43%	–15.84%	–12.49%	–7.49%

Source: own design, based on own calculations

Projected onto the entire loan portfolio, at a confidence level of 95% the market value of assets may potentially decrease by 7.28%, which – ignoring the change in the value of liabilities – results in the change in the market value of capital, and thus the change in the value of economic capital. As a result of this, the capital requirement of the hypothetical loan portfolio due to the prepayment option component of interest rate risk in the banking book, at a 95% confidence level, is 7.28% projected onto the entire exposure.

The distribution in the case of a rising interest rate environment is shown in *Figure 6.* The key statistics relating to the effect on individual loan portfolio elements and the loan portfolio as a whole are contained in *Table 17.* In the case of rising interest rates, similarly to the income-based approach, prepayment likewise has an impact, its scale being around one third of the result obtained in the case of declining interest rates.

Figure 6
Distribution of economic capital effect, determined based on change in discounted cash flow across entire loan portfolio*



Note: *broken line 99% confidence level, solid line 95% confidence level
Source: own design, based on own calculations

Table 17

Statistics of economic capital effect – ascending yield curve

Sub-portfolios	1.	2.	3.	4.	5.	Overall effect
Coupon rate	*4%*	*5%*	*6%*	*7%*	*8%*	
Remaining term to maturity	*10*	*5*	*6*	*7*	*4*	
Average	0.00%	–0.01%	–0.14%	–3.42%	–6.52%	–2.02%
Spread	0.01%	0.04%	0.20%	0.56%	0.52%	0.15%
95% confidence level	0.00%	–0.07%	–0.56%	–4.33%	–7.39%	–2.28%
99% confidence level	0.00%	–0.21%	–0.88%	–4.70%	–7.71%	–2.38%

Source: own design, based on own calculations

3.5 Application of stressed interest rate environment

In guidelines on the management of interest rate risk in the banking book – issued by the European Banking Authority (EBA) in May 2015,[7] and reviewing the earlier guidelines formulated by the one-time Committee of European Banking Supervisors (CEBS) – the application of interest rate shocks appears as an emphasised goal for evaluating the degree of exposure to interest rate risk. The guidelines state that institutions are obligated to assess the sensitivity of economic capital value and net interest income to potential changes in the yield curve, including parallel shifts and changes in shape. In addition, they are obligated to measure the effect of interest rate shocks on the value of economic capital, to a degree prescribed by the regulator. The size of the regulatory interest rate shock is a sudden parallel shift in the yield curve of +/– 200 basis points. If this is lower than the currently observed change in interest rate levels, then the 99% VaR of daily changes in interest rates[8] must be taken as the basis for calculations.

Table 18 features statistics calculated for stressed points on the forint yield curve, highlighting the one-year VaR with a 99% confidence level, expressed in basis points. I determined the degree of the stress shift in interest rates applied

7 EBA/GL/2015/08, Guidelines on the management of interest rate risk arising from non-trading activities

8 The 99th percentile of daily interest rate changes calculated looking back at a five-year period, annualized.

to the hypothetical portfolios using the one-year VaR, determined on the basis of the five-year data belonging to the three-month forint yield curve point (as the short interest approach), as shown in *Table 19*.

Table 18
VaR values at stressed points on the forint yield curve

Forint yield curve points	M3	M6	M12	Y3	Y5	Y10
Average	–0.13%	–0.13%	–0.13%	–0.09%	–0.07%	–0.05%
Spread	1.61%	1.40%	1.40%	1.94%	2.00%	1.88%
Annual spread	25.42%	22.21%	22.07%	30.74%	31.59%	29.76%
VaR (1 day, %)	3.74%	3.27%	3.25%	4.52%	4.65%	4.38%
VaR (1 year, %)	59.13%	51.67%	51.34%	71.52%	73.50%	69.22%
VaR (1 year, bp)	0.60%	0.53%	0.51%	1.51%	2.19%	2.69%

Source: MÁK, own design, based on own calculations

Table 19
Interest rate stress scenarios in the modelling

Stress scenarios	Declining yield curve	Ascending yield curve
r_0	6%	5%
b	4%	7%
VaR (1 year, bp)	3.55%	2.96%

Source: own design, based on own calculations

Assuming a declining interest rate environment, I examined the effect of the shift on the stress side with parallel downward and upward 355 basis-point shifts in the simulated yield curves. Assuming a downward shift in the yield curve, the income effect is significant for all sub-portfolios, while the effect projected onto the entire portfolio is close to four times greater compared to the results of the non-stressed interest rate environment. Sudden change in the interest rate environment also explains the significant effect in the case of individual sub-portfolios, as the interest rate declines abruptly from the 6% level applied in the model to below 4%, increasing the incidence of prepayment in all sub-portfolios. In the case of an upward shift in the yield curve, the prepayment inclination decreases considerably, so that its effect is much lower compared to the non-stressed environment. We may regard a downward shift in a declining yield curve as a genuine stress scenario. The economic capital-based effect produces a four-fold result compared to the non-stressed environment.

Assuming an ascending interest rate environment, I examined the effect of the shift on the stress side with parallel downward and upward 296 basis-point shifts in the simulated yield curves. The downward shift of the yield curve, similarly to the results obtained in a declining interest rate environment, is more than five times greater than the results observed in a non-stressed interest rate environment. The results obtained are half of the stress results of the declining interest rate environment. The rise in the yield curve likewise does not cause any substantial stress scenario. The tendency is similar with economic capital-based results, as in the case of a declining yield curve we obtain six times greater potential losses compared to the non-stressed interest rate environment.

4. SUMMARY

The model examines the effect of the optimal prepayment option on the cash flow of bank portfolios and the value of economic capital. Based on the model's results, it can clearly be stated that – depending on the composition of the bank portfolio (interest rate level, maturity) – the prepayment option can have a significant impact on both short-term, one-year total interest income and, via changing cash flows, on the bank portfolio's discounted value and through this the value of economic capital. The results are largely influenced by the portfolio's interest rate structure (coupon rates) and how it relates to changes occurring in the interest rate environment (declining/ascending yield curve). In the case of a declining yield curve, the effect makes itself felt more forcefully, while in the case of an ascending yield curve the effect of the refinancing incentive is likewise valid, albeit less forcefully. The model did not extend to an examination of the behaviour patterns of individual borrowers, since this could only be carried out based on genuine bank data.

The inclusion of prepayment charges in the model shifts the results in an interesting direction, resulting in a more forceful impact on interest income than in the version without such charges. Intuitively we would think that this cost element significantly restricts utilisation of the prepayment option, thereby reducing its impact. The lesson that can be drawn from the results, however, is that the cost level set in the model was too low to invalidate most optimal decisions so that a decrease in the number of incidences of prepayment would compensate for the effect on interest income brought about by redemption at a lower interest level. The applied cost level, however, cannot be much higher than the fair price that corresponds to the cost regimen of internal bank administrative processes relating to prepayment, which does not compensate for even a fraction of the shortfall in interest income.

Differences in sales channels and their varying incentive mechanisms were not taken into account in the model, so that I am unable to draw any conclusions about the impact of these on prepayment based on the model. Utilisation of the brokering sales channel may introduce a strong distorting effect into the system, since it does not necessarily support optimal decisions for customers, or because it incorporates further cost elements which partially burden the customer but which may also have a significant portfolio effect on the result side.

Based on the results of the model, it can be asserted that the interest income effect, depending on assumptions made about the interest rate environment, can be very considerable with respect to expected interest income both in the short term and throughout the loan duration. With regard to the income effect, I do not take into account dynamic changes in the balance sheet, such as the effect of multiple prepayments occurring, the possible repricing of the portfolio or an increase in volumes. For this reason, the obtained results are only suitable for revealing the effect of optimal prepayment in the short term, giving an indication of the interest income effect within one year and determining a potential maximum. Since the income effect does not take the time value of money into account, this method is unsuitable for quantifying long-term effects, but is an appropriate tool for managing short-term income.

Change in capital value I attribute in the model to the result of change in the present value of cash flows. This approach also allows long-term effects to be quantified, since it determines a theoretical bond price, as well as any change occurring therein. Methodologically, this approach fits into the logic of determining the capital requirement, on which long-term capital management decisions can be based.

I carried out the model calculations assuming two different interest rate environments, following a declining and an ascending yield curve. With regard to the composition of the portfolio, the individual sub-portfolios appear in the loan portfolio with the same weight in capital value. In the case of a declining yield curve, the interest income effect makes itself felt more forcefully, while in the case of an ascending yield curve the effect of the refinancing incentive is likewise valid, albeit less forcefully. The effect applies in varying degrees to the individual sub-portfolio elements. As the coupon rate increases, the interest income effect becomes increasingly strong, both in the case of a declining or an ascending yield curve.

The composition effect was not included separately in the model, as this would alter the result in a linear way by modifying the proportions. Using this factor would make sense if correlations between the individual sub-portfolios were to be included in the modelling, which could be built into the model by correlating the random numbers used for the simulation.

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