IMPLIED CORRELATION ON THE FOREIGN EXCHANGE MARKET

A review of the literature

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The overriding importance of correlation on the capital market is public knowledge. Scholars initially focused on the study of rates and returns, while later the theories of option analysis and time-series-based models brought volatility to the fore. Nowadays many portfolio management strategies, derivative market speculation and hedging strategies hinge on the accurate estimation of correlation. Implied correlation is a kind of market expectation regarding the future correlation of two foreign exchange rates, which can be calculated from implied volatility based on option prices. This paper presents the algebraic and geometric interpretation of implied correlation, the main time-series-based models, as well as the results of empirical tests. The expansion of time-series-based models can provide additional information regarding the expected realised correlations.

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1. METHODOLOGY

1.1. Implied correlation

Implied correlation can be calculated when two assets are connected by a third, or when one asset is part of the other asset. An example of the first case is the foreign exchange market where, based on the threefold determination of currency pairs, the arbitrage-free spot rate can be calculated from the two currency pairs. An example of the second case is the relationship between stock exchange indices and the constituent individual stocks, where the variance of an index is the weighted variance and covariance of the constituent individual stocks.[1] Implied correlation is also calculated in the case of credit derivatives where a lot depends on the estimation of expected correlations.[2] There is a relation known on the foreign exchange market whereby the rates of the three foreign exchange pairs (derived from the three foreign currencies) are interdependent in every moment due to no-arbitration. Let us call this relation the threefold determination of foreign

1 For more details see: Skintzi, Vasiliki D. – Refenes, Apostolos N. (2005)
2 For more details see: Hull, J. C. – White, A. D. (2006)

exchange rates. This means in effect that the rate of one currency pair can, in any given moment, be expressed as the product or quotient of the other two rates. For example, $\mathrm{HUF/USD} = \frac{\mathrm{HUF/EUR}}{\mathrm{USD/EUR}}$, that is, the price of USD expressed in HUF is always equal to the quotient of EUR expressed in HUF and EUR expressed in USD. If that would not be the case, the arbitrageurs would capitalise on the dichotomy and make a risk-free profit. This relation suggests that there also has to be a connection between the implied volatilities calculable from option transactions involving currency pairs.[3] The connection can be established by means of the log returns of foreign currencies, and according to the following correlation can be established: *Campa – Kevin Chang* (1998).

Let $S_{1,}$ S_2 and S_3 be the three foreign exchange rates derived from the three currencies. The authors use the designation S_1 = , S_2 = i and S_3 = . Let S_3 be expressed by the S_2/S_1 quotient, that is. $\mathrm{JPY/DEM} = \frac{\mathrm{JPY/USD}}{\mathrm{DEM/USD}}$ Also, let s_1, s_2 and s_3 be the log return on day t of the three foreign exchange rates, which, relative to the k^th^ currency pair, can be calculated as follows: $s_k = \ln\left(\frac{S_t}{S_{t-1}}\right)$. In this case $s_3 = s_2 - s_1$, which can be derived from the logarithms of the rates.[4] Also, the variance of the third probability variable (itself derived from the difference of the two correlated probability variables) can be described as follows (*Medvegyev – Száz*, 2010, p. 262.):

$$\sigma_{3,t,T}^2 = \sigma_{1,t,T}^2 + \sigma_{2,t,T}^2 - 2\rho_{t,T}\sigma_{1,t,T}\sigma_{2,t,T} \qquad (1)$$

where *t* is the first, and *T* the last, moment of observation; and *ρ* is the correlation of the log return of the first and second currency rate for the given period. The correlation can be expressed as follows:

$$\rho_{t,T} = \frac{\sigma_{1,t,T}^2 + \sigma_{2,t,T}^2 - \sigma_{3,t,T}^2}{2\sigma_{1,t,T}\sigma_{2,t,T}} \qquad (2)$$

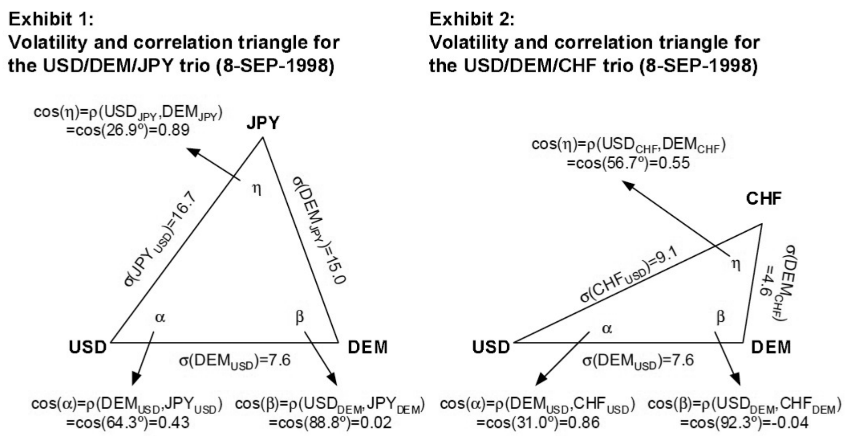
As it can be seen, the distributions and variances of the individual probability variables (currency log returns) are on the right side of the equation. The question is where these volatilities "have come from"? Possibly, the (implied) volatilities – back-calculated from the option market-making – are taken as a basis, in which case we calculate implied correlation. Another solution would be to use some kind of a time-series-based volatility estimation whose internationally accepted methodology involves a GARCH (generalised autoregressive conditional heteroscedasticity) model. The resulting correlation coefficient is called GARCH-based correlation.

3 For more details see: Zsembery (2003)

4 $\ln(S_{3,t}/S_{3,t-1})=\ln(S_{3,t})-\ln(S_{3,t-1})=\ln(S_{2,t}/S_{1,t})-\ln(S_{2,t-1}/S_{1,t-1})=\ln(S_{2,t})-\ln(S_{2,t-1})-[\ln(S_{1,t})-\ln(S_{1,t-1})]=\ln(S_{2,t}/S_{2,t-1})-\ln(S_{1,t}/S_{1,t-1})=s_2-s_1$.

The threefold determination of currency pairs and the consequent correlation determination is interpreted spectacularly in a paper by *Walter, Ch. – Lopez, J. A.* (1999): *“The Shape of Things in a Currency Trio”*. Implied correlation can be visualised by means of trigonometry:

Fig. 1



Source: Walter – Lopez (1999), p. 3.

The side lengths of the triangle are proportional to the implied volatilities of the currency pair consisting of the two foreign currencies, while the cosine of the angle they enclose correspond to the implied correlation of the two currency pairs. Apply the law of cosines[5]known from trigonometry to implied volatility and implied correlations:

$$\sigma_c^2 = \sigma_a^2 + \sigma_b^2 - 2\rho_{ab}\sigma_a\sigma_b \tag{3}$$

and the result will be identical with equation (1). In the triangle on the left the implied volatility of USD/DEM and USD/JPY will be the c and b sides of the triangle, and the value of implied correlation between the two currency pairs the cosine of angle α. It should be noted that because the starting point is the variance of log returns, the quotation of currency pairs is indifferent; for example, $\sigma_{USD/DEM} = \sigma_{DEM/USD}$. The shape of the triangle allows for the following assumptions:

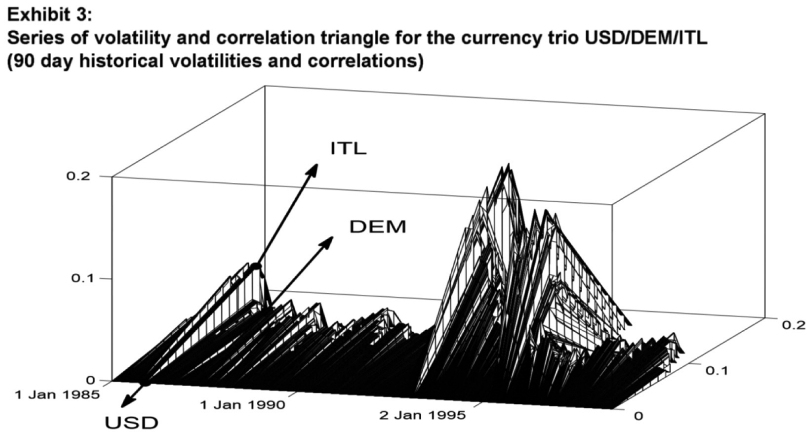
1. A large side length indicates high implied volatility for the given currency pair.
2. A small enclosed angle indicates high implied correlation relative to the two currency pairs.

5 $c^2=a^2+b^2-2ab\cos(\gamma)$

3. An angle larger than 90 degrees indicates negative implied correlation between two currency pairs.

The authors also point out that the geometric depiction of the correlations between currency trios can be interpreted not only in the context of a static approach, but also in a dynamic one. The example they cite is the development of distribution correlation over 90 days for the currency pairs involving USD, DEM and ITL. The examined period was between 1 January 1985 and 30 June 1998:

Fig. 2



Source: *Walter – Lopez* (1999), p. 5.

Clearly, the relationship between the previously highly correlated currency pairs ITL/USD and DEM/USD changed in 1992, and the volatility of the ITL grew. This is confirmed by the lengthening of the ITL sides of the triangles and the increase of the angle at the USD corner, attesting to the decrease of correlation between the ITL/USD and DEM/USD pairs. The change was occasioned by Italy's withdrawal from the exchange rate mechanism of the European Monetary System in September 1992.

Importantly, while currency-trio-based volatility–correlation triangles are characterised by the fact that the three volatilities (that is, the side lengths) determine the three correlations, the same cannot be said about the opposite. (If, for example, for given correlations, we proportionally increase every side length, the volatilities will increase while the correlations remain unchanged.) The study goes on to establish that the three implied correlations between the three currency pairs have to meet the condition that the angles of a triangle add up to 180 degrees (**π**):

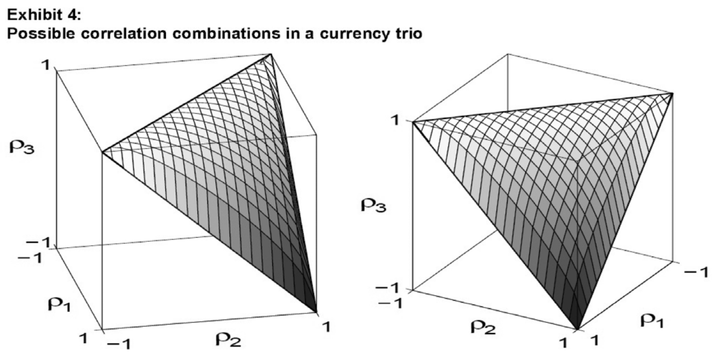
$$arccos(\rho_1) + arccos(\rho_2) + arccos(\rho_3) = \pi \tag{4}$$

Accordingly, a third correlation automatically follows from two known implied correlations:

$$\rho_3 = \cos\,(\pi\text{-}\ arccos(\rho_1) - arccos(\rho_2)) \tag{5}$$

The possible correlation combinations of currency trios can now be illustrated in a three-dimensional array; specifically a data cube having sides that span a -1;1 interval:

Fig. 3



Source: *Walter – Lopez* (1999), p. 6

At this point some special cases need to be considered. When two correlations are 0, the third will be 1 or –1 which cannot be interpreted geometrically, since no triangle exists with two right angles. This means that two currency pairs do not correlate with any other currency pair, while the third is in perfect linear relationship with one of them. Although the example is fictitious, if the CHF/EUR rate was fixed at the 1.20 level, the USD/EUR and CHF/USD rates would be negatively correlated, since

$$\mathrm{USD/EUR} = \frac{\mathrm{CHF/EUR}}{\mathrm{CHF/USD}}, \quad \text{thus } \mathrm{USD/EUR} = \mathrm{CHF/EUR} \times \mathrm{USD/CHF}$$

and thereby $\mathrm{USD/EUR} = 1{,}2 \times \mathrm{USD/CHF}$.

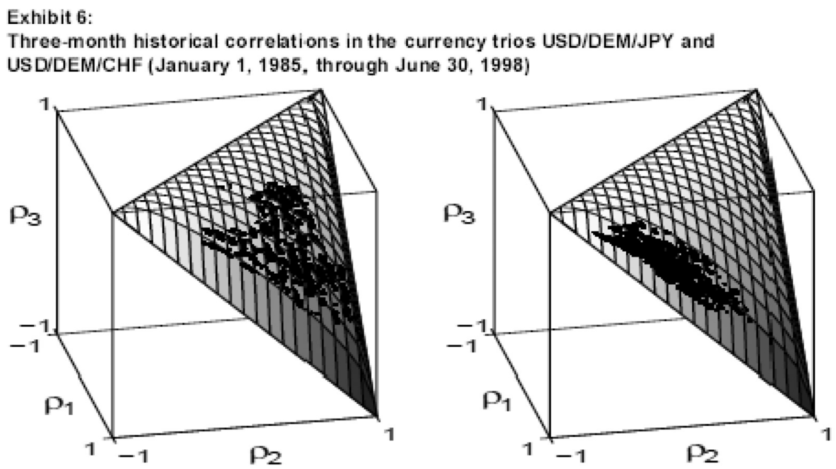
The example raises the question whether or not, if the CHF/EUR rate is fixed, the option market prices volatility, that is, is there an option market? Assuming market players are certain that the rate will remain permanently fixed, it is unlikely anyone will buy options; consequently, there will not be a market for options, an implied volatility will be zero. Accordingly, the triangular relationship between volatilities and correlations cannot be interpreted. However, if there is an option market (because certain players expect the fixing of the rate to be cancelled), some volatility can be expected and implied correlation will likely be higher than zero.

The correlation of currency pairs must also meet the condition of the positive semi-definiteness of correlation matrices. For 3 × 3 correlation matrices this condition is as follows:

$$\rho_1{}^2 + \rho_2{}^2 + \rho_3{}^2 - 2\rho_1\rho_2\rho_3 \leq 1 \tag{6}$$

In summary, currency-market implied correlation matrices are part of a subset of possible three-variable correlation matrices whose entries conform to equation (4). The joint visualisation of realised historical and possible correlations affords an interesting and spectacular opportunity for analysis, which the authors explored for the USD/DEM/JPY and USD/DEM/CHF currency trios. The examined period was between 1 January 1985 and 30 June 1998, again broken down to 90-day periods, as follows:

Fig. 4



Source: Walter – Lopez (1999), p. 8

For JPY and CHF, correlations with the other two currencies are marked in the ρ_1 and ρ_2 dimensions. It can be seen that in the case of CHF correlation is more stable than for JPY, that is, realised points are less scattered. That is particularly true for CHF along ρ_1 which is the correlation between log returns on USD/DEM and USD/CHF, that is, essentially it indicates correlation between CHF and DEM.

1.2. Time-series-based correlation

In their paper "The Forecasting Ability of Correlations Implied in Foreign Exchange Options", *Campa* and *Kevin Chang* (1998) provide an excellent summary of time-series-based models of correlation. These models include historical correlation, exponentially weighted moving average (EWMA) and the GARCH model. For the historical correlation model, the authors regarded realised correlations of the past month and of the past three months as the forecast for the future:

$$\rho_{a-b,t-T}^{HIST} = \frac{\sum_{j=1}^{n}(r_{a,t} - \overline{r_a})\,(r_{b,t} - \overline{r_b})}{\sqrt{\sum_{j=1}^{n}(r_{a,t} - \overline{r_a})^2}\sqrt{\sum_{j=1}^{n}(r_{b,t} - \overline{r_b})^2}} \tag{7}$$

where $r_{a,t}$ and $r_{b,t}$ are the log returns of a and b currency rates for day t, and $\overline{r}_{a,t}$ and $\overline{r}_{b,t}$ are the average daily log returns characteristic of the given period.
The EWMA is a model developed by JP Morgan RiskMetrics™ for the forecast of correlation (JP_Morgan, 1996), in which the data of the recent past are more heavily weighted, and weighting exponentially decreases as follows:

$$\rho_{a-b,t-T}^{EWMA} = \left[\frac{1}{\sum_{j=1}^{n}\lambda^j}\right]\sum_{j=1}^{n}\lambda^j \rho_{a-b,t-j-T,T} \tag{8/a}$$

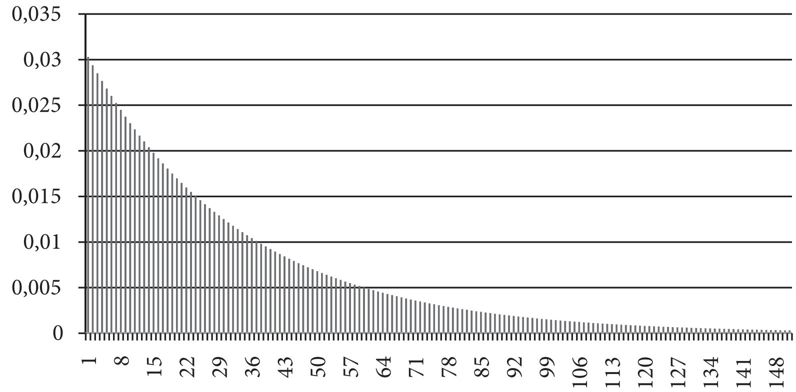
which can be transcribed as follows:

$$\rho_{a-b,t-T}^{EWMA} = \frac{\sum_{j=1}^{n}\lambda^j(r_{a,t} - \overline{r_a})\,(r_{b,t} - \overline{r_b})}{\sqrt{\sum_{j=1}^{n}\lambda^j(r_{a,t} - \overline{r_a})^2}\sqrt{\sum_{j=1}^{n}\lambda^j(r_{b,t} - \overline{r_b})^2}} \tag{8/b}$$

The λ parameter in the formula determines the pace of the decrease of the weights. In keeping with experience reported in the literature,[6] the authors set parameter λ at 0.97, and made n=151 days as the optimum value of the history.

6 For more details see: (JP_Morgan, 1996), p. 100

Fig. 5.
Weights (λ = 0,97; n = 151)



The forecasts of both historical and EWMA correlations are made on the basis of observed correlations. The generalised autoregressive conditional heteroscedasticity (GARCH) model is widely used for modelling volatility changing over time. In calculating GARCH-based volatility, the authors back-calculated correlation by modelling the conditional variance of the daily log returns of individual currency pairs, and the conditional covariance of the log returns of the currency pairs. The modelling of the conditional variance of individual rates can be performed independently or by means of a two-variable GARCH model. In the case of a single variable, the correlation for each currency pair can be deduced from the variance of the three currency pairs, which can take any value between -1 and 1. However, in the two-variable GARCH model the variances of the individual currency pairs and the covariance between the, are simultaneously estimated; that is, a relationship is already assumed between the individual currency pairs. In this case, the conditional variance of the individual currency pairs not only depends on its own previous values, but also on the lagged values of the returns on the other currency pair. On the basis of a Lagrange multiplier test the authors decided not to use the single-variable method and proceeded to use the two-variable method thereafter. The general model of the two-variable GARCH(p, q) is as follows:

$$s_t = \bar{s} + \epsilon_t$$

$$vech(H_t) = c + \sum_{i=1}^{q} A_i vech(\epsilon_{t-i}\epsilon'_{t-i}) + \sum_{j=1}^{p} B_j vech(H_{t-j}) \tag{9}$$

$$\epsilon_t | \psi_{t-1} \sim N(0, H_t)$$

where s_t is the 2×1 vector of the two currency pairs for day t; $\bar{s}$ is the 2×1 vector containing the averages of the daily log returns; and ε_t is the 2×1 error vector. The vech operator forms a column vector from the lower triangle of the symmetrical matrix by "stacking" the columns of the triangular matrix. H_t is the 2 × 2 covariance matrix; therefore in this case vech (H_t) is a 3 × 1 column vector which contains the variance of the log returns of the first currency pair, the covariance of the log returns of the two currency pairs, and the variance of the log returns of the second currency pair. ψ_{t-1} is the information available in the t–1st moment in time. The C 3 × 1 vector contains the constants which, in the GARCH model, give the long-term equilibrium value of the variable (variance and covariance). ($\varepsilon_{t-1}\varepsilon_{t-1}$') is the 2 × 2 matrix that contains the dyadic products of errors, in the diagonals of which are the squared errors of the log returns of the two currency pairs, and outside of the diagonals are the cross products of the errors. Semi-vectorisation of these will transform them into a 3 × 1 column vector. This column vector is multiplied from the left by the A_i 3 × 3 matrix, which is the ARCH component of the GARCH model (containing the coefficients relative to the error terms), and which is 3 × 3 because the error terms of the other two variables are also included in estimating both the variance and the covariance of the currency pairs. The general formula adds up ARCH(q) components (it contains a q number of lagged error terms), while it contains a GARCH(p) component whose coefficients are included in the B_i 3 × 3 matrix. Compared to this general formula, matrices A_i and B_i are transformed into diagonal matrices, that is, only its own lagged values will be included in the estimation of variance/covariance. This means that, compared to the 23 parameters in the general formula, only 11 need to be estimated. Estimation of parameters is achieved by maximising the log-likelihood function below:

$$L_t(\theta) = -\frac{N}{2}log2\pi - \frac{1}{2}log|H_t(\theta)| - \frac{1}{2}\epsilon_t(\theta)'H_t^{-1}(\theta)\epsilon_t(\theta) \tag{10}$$

where θ is the vector containing all of the parameters. Since, based on the results published in the literature, the GARCH(1,1) specification works well with daily currency returns, the authors chose to go along with it, and the sums could be removed from equation (9). The individual rows of the resulting vector equation can be written up as follows:

$$\begin{aligned} s_t &= \bar{s} + \epsilon_t \\ h_{ij,t} &= c_{ij} + \propto_{ij} \epsilon_{i,t-1}\epsilon_{j,t-1} + \beta_{ij}h_{ij,t-1} \\ \epsilon_t|\psi_{t-1} &\sim N(0, H_t) \end{aligned} \tag{11}$$

The same written up with matrices and vectors:

$$\begin{bmatrix} h_{11,t+1} \\ h_{12,t+1} \\ h_{22,t+1} \end{bmatrix} = \begin{bmatrix} \omega_{11} \\ \omega_{12} \\ \omega_{22} \end{bmatrix} + \begin{bmatrix} \alpha_{11} & 0 & 0 \\ 0 & \alpha_{12} & 0 \\ 0 & 0 & \alpha_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t}^2 \\ \varepsilon_{1,t}\varepsilon_{2,t} \\ \varepsilon_{2,t}^2 \end{bmatrix} + \begin{bmatrix} \beta_{11} & 0 & 0 \\ 0 & \beta_{12} & 0 \\ 0 & 0 & \beta_{22} \end{bmatrix} \begin{bmatrix} h_{11,t} \\ h_{12,t} \\ h_{22,t} \end{bmatrix} \tag{12}$$

Since the estimation of the GARCH model based on daily data provides a one-day forecast, to achieve one- and three-month estimations, the n^{th} day estimations are recursively substituted, and the squared errors correspond to the estimated variances:

$$\bar{h}_{ij,t+1} = \hat{c}_{ij} + \hat{\alpha}_{ij}\ \epsilon_{i,t}\epsilon_{j,t} + \hat{\beta}_{ij}\bar{h}_{ij,t} \tag{13}$$

where $\bar{h}$ ist he estimated variance (covariance). Because variances are additive in time, the variance estimation for day n is simply the sum of the estimated values. The authors discuss three approaches for estimating the parameters with regard to the sampling. The first approach is parameter estimation based on the entire sample, which will produce good results if the sample is stable in time. The second possibility is the so-called rolling GARCH, meaning that the estimation of the parameters is only performed on the data of the past period of fixed length. The combination of the two is called updating GARCH which uses all data available up to the given moment in time. The authors are in favour of the rolling GARCH because, due to instable coefficients, it provides better estimations during changeable periods. They estimated the parameters of the GARCH model on the basis of a 1000-day (4-year) sample.

2. EMPIRICAL RESULTS

2.1. Campa – Kevin Chang (1998)

Campa and *Kevin Chang* processed the data of 1 600 trading days between 3 January 1989 and 23 May 1995 for the cross rates of DEM/USD, JPY/USD and JPY/DEM (*Campa – Kevin Chang*, 1998).Implied volatilities were calculated on the basis of one- and three-month forward ATM straddle options observed on the given day. Accordingly, one-month implied volatilities played a role in forecasting one-month correlations, and three-months volatilities in three-month correlations. For the rolling GARCH forecast the authors used the data of one thousand trading days, that is, January 1989 estimations also drew on trading data from early 1985. The authors assessed the appropriateness of the forecasts gained by different methods in three different ways.

1. root-mean-square error (RMSE)
2. running a regression between realised correlations and the individual forecasts and comparing their R^2 (that is the proportion of explained variance)
3. finally they ran a multi-variable regression which included several parallel methods as explanatory variables In this case they employed a Wald test to decide whether adding other variables to implied correlation significantly increased the predictive power of the model.

The root-mean-square error (RMSE) method yielded the following results:

Table 1
RMSE of 1–3-month forecasts (Campa – Kevin Chang)

Method	RMSE 1 month	RMSE 3 months
Historical	0.2215	0.1472
Implied	0.1702	0.1377
EWMA	0.182	0.1639
GARCH(1,1)	0.2082	0.1691

Source: Campa – Kevin Chang (1998), p. 27.

As it can be seen, the implied correlation has a lower root-mean-square error in both the one- and three-month time span, that is, it affords a more accurate forecast. Furthermore, the predictive power of all methods improves as the time span is increased, that is, in the long run correlation becomes more stable and is characterised by return to the average.

Next, they estimated regression. In doing so, they explained realised correlation first by means of the individual methods, then their combination and a single constant. It should be noted that the standard errors of the parameters produced by regression estimation require correction (variance is lower than reality) due to the overlapping periods of observation.

Table 2
Regression coefficients of 1–3-month forecasts (Campa– Kevin Chang)

		1 month			
Constant	**Implied**	**Historical**	**GARCH(1,1)**	**EWMA**	**R^2**
0.151*	0.741*				0.29
(0.060)	(0.090)				
0.361*		0.369*			0.13
(0.056)		(0.089)			
0.402*			0.302*		0.08
(0.129)			(0.052)		
0.206*				0.643*	0.21
(0.076)				(0.122)	
0.152**	0.762*	−0.023			0.29
(0.060)	(0.115)	(0.099)			
0.252**	0.823*		−0.0263		0.31
(0.106)	(0.090)		(0.193)		
0.136**	0.650*			0.117	0.29
(0.068)	(0.117)			(0.148)	
0.235**	0.749*	−0.052	−0.268	0.161	0.31
(0.112)	(0.138)	(0.104)	(0.197)	(0.158)	

		3 months			
Constant	**Implied**	**Historical**	**GARCH(1.1)**	**EWMA**	**R^2**
0.174**	0.678*				0.30
(0.082)	(0.123)				
0.250*		0.560*			0.31
(0.076)		(0.127)			
0.415**			0.268		0.03
(0.187)			(0.331)		
0.318*				0.440**	0.15
(0.103)				(0.182)	
0.168**	0.369*	0.327**			0.34
(0.080)	(0.126)	(0.152)			
0.278	0.800*		−0.309		0.32
(0.145)	(0.161)		(0.324)		
0.177**	0.710*			−0.039	0.3
(0.090)	(0.194)			(0.253)	
0.293**	0.551*	0.805*	−0.221	−0.666**	0.43
(0.116)	(0.151)	(0.217)	(0.246)	(0.305)	

Notes: *differs from zero at a 1% significance level
**differs from zero at a 5% significance level
Source: Campa – Kevin Chang (1998), pp. 28–29.

All of the resulting coefficients are lower than one and all coefficients are significantly not zero, except with the GARCH(1,1)–3-month time-span combination. The corrected R^2, indicating the general goodness of the model, is the highest for implied correlation in the one-month time-span.

2.2. Walter – Lopez (2000)

Walter and *Lopez* (2000) examined the predictive power of implied correlation for the USD-DEM–JPY currency trio (1679 observations in 1990–1997) and the USD–DEM–CHF currency trio (910 observations in 1993–1997). Like in (*Campa–Kevin Chang,* 1998), the source of implied volatility consisted of forward ATM straddle options. The authors came to the conclusion that for the USD–DEM–JPY currency trio implied correlation afforded a distorted estimation of future realised correlation, but in many cases, in comparison with models based on historical data, it provided additional information with regard to future realised correlation. The authors' results do not confirm the results of (*Campa–Kevin Chang,* 1998) with respect to the superiority of implied correlation over a one–month period for the USD–DEM–JPY currency trio:

Table 3
Forecast errors in 1-month period for USD–DEM–JPY (Walter – Lopez)

	ρ(DEM/USD; JPY/USD)		ρ(USD/DEM; JPY/DEM)		ρ(USD/JPY; DEM/JPY)	
Method	**MSE**	**RMSE**	**MSE**	**RMSE**	**MSE**	**RMSE**
Implied	0.048*	0.182	-0.024*	0.233	-0.080*	0.297
Historical (20 days)	0.003	0.218	0.002	0.281	-0.001	0.323
Historical (60 days)	0.016	0.187	-0.009	0.262	-0.028**	0.295
Historical (120 days)	0.026*	0.189	-0.013	0.279	-0.043*	0.310
EWMA (0.94)	0.009	0.188	-0.005	0.243	-0.016	0.288
EWMA (0.97)	0.018	0.177	-0.009	0.241	-0.034**	0.279
EWMA (0.99)	0.031*	0.190	-0.010	0.257	-0.069*	0.319
GARCH	0.012*	0.17	0.014*	0.238	-0.017*	0.278

Notes: *differs from zero at a 1% significance level
**differs from zero at a 5% significance level
Source: Walter – Lopez (2000), p. 33.

Implied correlation did not prove to be superior to the time-series-based methods for the USD–DEM–CHF currency trio either:

Table 4
Forecast errors in 1-month period for USD–DEM–CHF (Walter – Lopez)

	ρ(DEM/USD; CHF/USD)		ρ(USD/DEM; CHF/DEM)		ρ(USD/CHF; DEM/CHF)	
Method	**MSE**	**RMSE**	**MSE**	**RMSE**	**MSE**	**RMSE**
Implied	0.017*	0.041	–0.232*	0.363	0.151*	0.260
Historical (20 days)	–0.000	0.040	–0.018	0.332	0.020	0.259
Historical (60 days)	–0.002	0.040	–0.006	0.292	0.003	0.230
Historical (120 days)	–0.004	0.046	–0.002	0.294	0.001	0.227
EWMA (0.94)	–0.001	0.035	–0.010	0.293	0.008	0.226
EWMA (0.97)	–0.003	0.037	–0.005	0.278	0.004	0.215
EWMA (0.99)	–0.008	0.042	–0.011	0.278	0.014	0.207
GARCH	–0.005	0.043	–0.077	0.289	0.055*	0.195

Notes: *differs from zero at a 1% significance level
**differs from zero at a 5% significance level
Source: Walter – Lopez (2000), p. 35.

The authors highlight the fact that the lack of implied correlation's additional predictive power in the case of the CHF trio is due to the stability of correlations and volatilities in time.

In their final conclusion, they point out that the dubitable superiority of implied correlation can be put down to two possible factors. First, the specification of the Garman-Kohlhagen model, used for extrapolating implied volatility, is incorrect; and second, it is possible that certain option markets, based on which implied correlation is calculated, are not effective. The latter can be assumed mainly for the CHF trio where interbank trading is insignificant for certain option transactions. The same cannot be said about the JPY trio.

2.3. Castrén – Mazotta (2005)

Relatively recently, *Castrén* and *Mazotta* (2005) published results of their research of implied correlation. They examined the predictive power of implied correlation in the context of EUR, USD, GBP, JPY, PLN and CZK, comparing it to the GARCH and EWMA methods. The time-span is 1992–2004; pre-1999 data including DEM instead of EUR. Their results reveal how the predictive power of implied correlation differs for each currency pair and period, and that the best model combines the implied- and the time-series-based models. Their paper also examines the interventions of the Bank of EUR–USD–JPY currency trio revealed how, in the wake of the BOJ's purchases of USD against JPY, negative correlation increased in the context of JPY/EUR and USD/EUR. Including the intervention in the model explaining realised correlation, they achieved a significant negative coefficient:

Table 5
The coefficients of the regression model explaining the correlation of USD/EUR and JPY/EUR, with and without the intervention of the BOJ (Castrén–Mazotta)

	Entire sample			Sample after introduction of EUR		
Method	**Correlation**	**Intervention**	**R^2**	**Correlation**	**Intervention**	**R^2**
Implied	0.747*		0.205	0.924*		0.359
	(0.105)			(0.113)		
	0.745*	–0.28*	0.22	0.920*	–0.013*	0.365
	(0.067)	(0.053)		(0.069)	(0.044)	
Historical	0.564*		0.314	0.583*		0.343
	(0.053)			(0.076)		
	0.561*	–0.197*	0.326	0.581*	–0.013*	0.349
	(0.037)	(0.045)		(0.050)	(0.044)	
EWMA	0.874*		0.235	1.168*		0.382
	(0.112)			(0.143)		
	0.871*	–0.022*	0.242	1.163*	–0.011*	0.387
	(0.079)	(0.046)		(0.093)	(0.044)	
GARCH	0.858*		0.329	0.834*		0.362
	(0.066)			(0.094)		
	0.854*	–0.020*	0.341	0.832*	–0.014*	0.370
	(0.049)	(0.045)		(0.094)	(0.045)	

Note: *differs from zero at a 1% significance level
Source: Castrén – Mazotta (2005), p. 26

2.4. Esposito – Laruccia (1999)

Few research projects deal with further possibilities for implied correlation, for example, by charting the underlying structure of currency rates. *Esposito* and *Laruccia* (1999) explored the underlying forces affecting USD rates by analysing the main component applied to implied correlations, based on 1999 data. Due to the fact that not all of the currencies traded against the USD have a liquid option market, analysis was performed taking into consideration the main currencies only:

Table 6
Eigenvalues and eigenvectors of implied correlation matrix (Esposito–Laruccia)

Eigenvalues	**1**	**2**	**3**	**4**
value	2.6983	0.7084	0.574	0.0192
proportion of variance explained	0.6746	0.1771	0.1435	0.0048
cumulative proportion of explained variance	0.6746	0.8517	0.9952	1
Eigenvectors	**1**	**2**	**3**	**4**
EUR	0.5833	–0.0973	0.3365	0.7329
GBP	0.4325	–0.4755	0.764	–0.0565
JPY	0.3926	0.8637	0.3112	–0.0548
CHF	0.5645	–0.1358	–0.4541	–0.6758

Source: *Esposito – Laruccia* (1999), p. 17.

It can be seen that 67.46 per cent of the variance of USD rates is explained by the first main component in which all four currencies are weighted roughly identically, with a slight preponderance of EUR and CHF. The JPY and the GBP account for the second and third components.

3. SUMMARY AND FURTHER POSSIBLE RESEARCH QUESTIONS

Empirical results suggest that implied correlation is an unavoidable "player" of foreign-exchange-market correlation estimation. Although predictive accuracy varies for each currency trio and also in time, in many cases it carries useful information with regard to expected correlation and in many cases it proves to be superior to time-series-based correlation estimation.

The results presented above beg the question, how implied correlation would work, firstly, with fresher data and, secondly, with the HUF? As far as I am aware, no analysis has yet been performed for the EUR–USD–HUF currency trio, which is a gap that needs to be filled. Since, however, trading on the HUF market is, by international standards, very low, some fairly sweeping assumptions will have to be made concerning implied volatilities as market expectations. In a Hungarian National Bank working paper, *Gereben* and *Pintér* (2005) demonstrated that implied volatility in the one-month time scale affords useful information regarding future realised volatility, but the estimation will be distorted and cannot be regarded as superior to time-series-based estimations. As regards distortion it should be noted that if, in context of a given currency trio, the implied volatility of each currency pair is distorted in the same direction, the distortion of implied correlation will be lesser. In the period examined by the Hungarian National Bank there was a currency band in place on the HUF/EUR market, which in itself could have distorted implied volatilities.

There is, however, no need for administrative currency rate restriction. According to *Rebonato*'s famous saying, implied volatility is the wrong number to put in the wrong formula to obtain the right price (*Rebonato*, 2004). Because markets are neither complete nor information-efficient and implied scales often reflect disequilibrium of demand and supply (for example, the majority of the players in a stock market are in the long position and the demand for put options for the purpose of hedging is greater than for call options).

The management of volatility surface at all times questions the relevance of calculating implied correlation. Like volatility surfaces, correlations also constitute surfaces as a function of exercise price and maturity. The calculation and analysis of implied correlations derived from the price of 10- and/or 25-delta options affords an interesting comparison vis-à-vis the "usual" ATM options. That makes sense in the case of highly-traded currency trios, such as the EUR-USD-JPY.

It could also be interesting to examine the effect of central-bank interventions in the context of forecasting correlation, in particular concerning the cross rates of JPY and CHF.

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