THE AGGREGATION OF ECONOMIC CAPITAL REQUIREMENT WITH A CRISIS-DEPENDENT CORRECTION

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ABSTRACT

In comparison with the simple variance-covariance matrix-type approach to the aggregation of economic capital requirement there are other definable methodologies that take into consideration the current economic cycle. The disadvantage of simple methodologies is that they primarily draw on "peace-time" data and provide unreasonably high estimations for crisis periods. Since it is the inter-risk correlation effects that can be taken into consideration for the Hungarian market (the Hungarian National Bank refuses to recognise the diversification effect between major types of risk under the SREP), the present approach focuses on this detail. This analysis has led to the employment of the Markov regime-switching model which is able to achieve risk aggregation in a subtle way. The developed model was tested on a database that is simulated but corrected on the basis of the annually published data of certain institutions.[1]

JEL codes: C01, C13, C24, C51, G21, G28

Keywords: Capital requirement aggregation, Markov regime-switching model, credit risk

1. CLASSICAL AGGREGATION

Risk aggregation is a general corporate risk-management issue. It is about adequately measuring and summarising risks in an effort to produce data as relevant as possible for the management of risk. In addition to choosing an approach, however, diversification gains reducing capital requirement also assume a crucial role. These gains largely depend on measuring risk: different risk aggregation approaches can yield considerably different diversification gains. Being capital requirement, the different methodological solutions are hard to test by means of direct data; formalised deduction is therefore the way forward.

1 This study was created under the research and development project "Research into innovative mathematical models for measuring Basel bank risks and the quantification of capital requirements in the area of market, operational, liquidity and secondary risks; and the behaviour-based prediction of the price trends of financial products" (Project no. PIAC_13-1-2013-0073), funded within the framework of the New Széchenyi Plan.

The simplest methods of risk aggregation do not take into the equation the correlations between risk categories. This includes the simple addition of quantified capital requirements, an extreme case of variance-covariance method where the correlation coefficient across all risk categories is 1. This, too, simply generates aggregated capital requirement as the sum of capital requirements calculated for individual risk categories.

The variance-covariance method is a ubiquitous analytical technique used for the aggregation of risks. It enables the pooling of individual loss distributions in a combined loss distribution. The only factor required is the extent of subordination of losses, whose role is typically filled by the matrix of linear correlation coefficients. The lower the correlation coefficients are than the non-diagonal elements of the matrix, the higher diversification effect can be achieved. However, in most multi-dimensional distributions the correlation matrix – by summing up subordination in a single number – does not say enough about the interaction between two variables.

The following formula is used to calculate aggregated risk in this method:

$$R = \lambda \sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j cov(i,j)}$$

The formula applies the classical variance-covariance matrix for determining aggregated risk, where *cov(i,j)* is the covariance matrix, and w_i and w_j the relative weights of the individual elements.

Because the risk exposures are variables measured on a ratio scale, linear correlation is the most obvious method. The correlation matrix in this case consists of the linear correlation coefficients, as follows:

$$r(X,Y) = \frac{\sum d_{Xi} d_{Yi}}{\sqrt{\sum d_{Xi}^2 \sum d_{Yi}^2}} = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum (X_i - \bar{X})^2 \sum (Y_i - \bar{Y})^2}}$$

where $r(X,Y)$ is the correlation coefficient between X and Y risk categories, and d_{Xi} is the divergence of the i^{th} observation of risk category X from the average; that is, $d_{Xi}=(X_i-\bar{X})$. The correlation coefficient, by definition, can take a value between –1 and +1. A value of –1 indicates that the risk categories are moving deterministically in the opposite direction; a value of +1 means that they are moving deterministically in the same direction.

Typically, the methodology determines the matrix with the help of the available history; which, however, is not necessarily appropriate because when crisis hits, correlation relationships can change. One of the first discussions of the BIS stress tests calls for the survey of the change of this very correlation effect.

The variance-covariance method that is employed in calculating capital requirement applies a top-down approach, that is, risk capital requirements are quantified independently and are subsequently aggregated at the highest level (e.g. as credit risk). Each of the calculated capital requirement constitutes a scalar. Aggregation does not distinguish between capital requirement methods; IRB and standard portfolios are lumped under the same heading. The correlation matrix is a symmetrical n × n positive definite matrix (where n is the number of risk factors); its major axis contains 1s, so it is true that $x'Rx > 0$, ha $x \neq 0$.

Using the variance-covariance method, the aggregated capital requirement can be calculated as follows:

$$C_a = \sqrt{C_i^T \times R \times C_i}$$

where C_a is the aggregated capital requirement which, in a single figure, comprises both intra and inter-diversification effects; and C_i is the column vector of economic capital (EC) requirements calculated for the individual risk segments (portfolios or qualification categories), that is:

$$C_i^T = (EC_1, EC_2, \dots, EC_n)$$

In this analysis this is an initial model that treats the past as a whole, irrespective of whether the data come from a situation of crisis or situation of economic growth. Using this methodology in calculating aggregated capital requirement can, therefore, be problematic, since correlation relationships can change during a crisis. It is this that will be examined in the following.

2. DEVELOPMENT POSSIBILITIES FOR AGGREGATION METHODOLOGIES

Linear correlation is not always the right way to go in describing the subordination of several variables. The simple variance-covariance methodology employs the correlation matrix consisting of linear correlation coefficients for determining aggregated capital requirement.

The authors of this paper have proposed some alternative solutions (some of which have appeared in the literature) seeking to address the disadvantages of the linear relationship described in Chapter 1. Three methodologies were discussed: copula-based aggregation logic, scenario-based evaluation and a Markov regime-switching model.

Copulas are usually used to describe the structure of subordination of multiple parameters because they enable the break-down of the combined distribution of these variables into marginal distributions and the function describing the subordination thereof. *Sklar* proved this formally:

Where F is a d-dimensional function of distribution with F_1, ..., F_d marginal distribution values, there exists a C copula where $F(x_1,...,x_d) = C(F_1(x_i),...,F_d(x_d))$.

The copulas help to flexibly describe combined distributions in that any marginal distribution can be linked with any copula. Consequently, for example, the Gaussian copula cannot only be used where the marginal distribution is assumed to be Gaussian (normal), and empirical distribution can also be used as marginal distribution.

When using copulas, it is important to find the type that best suits the data set and establish the parameters that describe the dependence structure of the given type of copula. The next step is to estimate the dependence parameter, and a high-dimensional dependence structure can be modelled in a way that it closely resembles reality.

Widely used in general practice, single-parameter Archimedean copulas are easy to establish. Modelling increased same-direction movement between high PD and LGD or individual aggregated risk types, the Clayton copula is best used in the area of credit risk. In the case of market risk, it is worth choosing a copula that that envisages an increase in correlation whether change is negative or positive.

The quantification of losses of future states of the world involves the aggregation of scenarios describing the arrival of given events, making it possible to include specific special events in aggregating risk. This might be called scenario-based aggregation. To make this adequately accurate, it is necessary to chart possible events with great circumspection, including the movement of change in the risk categories, as well as the likelihood of the occurrence of specific scenarios. It is crucial to the method to identify the factors affecting risk exposure. That done, the given scenario (change of factor) can be simulated and risk exposures quantified; in other words, this method can determine the loss distribution functions for a given scenario.

The advantage of the scenario-based aggregation method is that it is consistent. Risk exposures for a given scenario are individually calculated and a wide range of cases are incorporated in the aggregation. Also, this methodology forces the institution using it to better understand risks and the factors influencing them. Its disadvantage is that it relies on considerable amount of assumptions, and consequently, the results too will include the effects of expert opinion, that is, the result will depend on the opinion of experts performing the evaluation.

The third option is the Markov regime-switching model. Historical capital requirement data show that institutions behave differently during crisis than they do in normal periods. This would suggest that different factors play a role in determining aggregated capital requirement, all of which can increase during a crisis period.

The powerful diversification effect that is characteristic of normal periods can decrease, necessitating a different aggregation model. Accordingly, an interim aggregation level needs to be set up and modelled in order help prepare for crisis-time capital requirements in a normal period. That is, with the help of an accurate capital requirement model, the institution can dampen the evident procyclical effect in capital requirement. This paper will, in the following, focus on what the authors believe to be the most useable and analytically estimable methodology.

3. MARKOV REGIME-SWITCHING MODELS

The Markov regime-switching models have been widely used in the literature of financial econometrics for modelling the behaviour of exchange rates and certain macroeconomic variables. *Engel* (1994) examined the predictability of rates with this type of model and established different trends for certain states of the economy. *Clarida*, *Sarno*, *Taylor* and *Valente* (2003) also examined several model specifications for forecasting exchange rate, including a Markov regime-switching model. The papers of *Frömmel*, *MacDonald*, *Menkhoff* (2005) and *Marsh* (2000) clearly demonstrate how the Markov regime-switching models were set up for prognosticating monetary exchange-rates in a way that they produced slightly more relevant predictions than random forecasts regarding the general trends of exchange rates.

Hamilton (1989) was a pioneer in successfully applying regime-switching models and analytically predicting macro-variables. Other models predicting macro-variables have been explored by *Blix* (1999), *Kim*, *Morley* and *Piger* (2005), and *Li*, *Lin* and *Hsiu-Hua* (2005).

At the same time this method has not yet been tested in the modelling of capital requirement and macro-economic variables. In light of the experiences of the financial crisis, the authors of this paper believe that this methodology can be used in modelling the aggregation of capital requirement. Firstly, it helps formalise the reasonable assumption that in times of a financial crisis or considerable market turbulence, the relationship between the quality of bank portfolios and macro-economic variables is different from "normal" periods, in particular when analysed at shorter (e.g. monthly) intervals. Secondly, the model uses relatively few parameters to describe the non-linearity of the relationship, and the durations of the various regimes do not have to be added externally as expert forecasts.

The simplest version of a Markov regime-switching model describes the relationship between two variables by means of two possible states, and gives the probability with which the process will shift from the one state to the other. Formally speaking, Let y_t be a given time series whose development in the different regimes (s_t=1,2) can be described by the following formula:

$$y_t = \mu_{s_t} + \sigma_{s_t}\varepsilon_t$$

where $\varepsilon_t \sim N(0,1)$, and

$$\mu_{st} = \beta'_{st}x_t \quad .$$

Here, s_t is a probability variable that shows which regime the process is in at t point in time. In the Markov regime-switching model it is assumed that s_t is the realisation of a two-state Markov chain, and consequently the transition probabilities describing the shifts of can be formalised as follows:

$$P(s_t = i|s_{t-1} = j, s_{t-2} = k, \dots, y_{t-1}, y_{t-2}, \dots) = P(s_t = i|s_{t-1} = j) := p_{ij}.$$

Due to the normal distribution of the random error, the probability of individual observations – based on and the information available up to point t (F_{t-1}) and including the values of the explanatory variables in the moment of point t (x_t) – is:

$$f(y_t|S_t, S_{t-1}, F_{t-1}) = \frac{1}{\sqrt{2\pi\sigma_{S_t}^2}} exp\left\{-\frac{\left[y_t - \mu_{S_t}\right]^2}{2\sigma_{S_t}^2}\right\}.$$

On the basis of the chain rule, it can be developed as follows:

$$f(y_t, S_t|F_{t-1}) = f(y_t|S_t, S_{t-1}, F_{t-1})P(S_t, S_{t-1}|F_{t-1}).$$

Again based on the chain rule and by applying the Markov property:

$$P(S_t, S_{t-1}|F_{t-1}) = P(S_t|S_{t-1}, F_{t-1})P(S_{t-1}|F_{t-1}) = P(S_t|S_{t-1})P(S_{t-1}|F_{t-1}).$$

On this basis, the log-likelihood function is:

$$L(\theta) = \sum_{t=1}^{T} log\left[\sum_{S_t=0}^{1}\sum_{S_{t-1}=0}^{1} f(y_t|S_t, S_{t-1}, F_{t-1})P(S_t|S_{t-1})P(S_{t-1}|F_{t-1})\right],$$

where θ is the estimated parameter vector whose elements are $\beta_0, \beta_1, \sigma_1, \sigma_2, p_{11}, p_{22}$.

Accordingly, calculating the likelihood function requires the conditional probability of the individual states (relevant to the given point in time, depending on the information available up till that point ($P(S_{t-1} \mid F_{t-1})$). This is established in the following multi-step process.

We assume that we know the pre-sample values of the two initial state probabilities, to which we assign a random value:

$P(S_0 = 1|F_0) = \pi$, és $P(S_0 = 2|F_0) = 1 - \pi$.

Next, for each observation, we can establish in two steps the probabilities in the likelihood function.

Step 1: we calculate the probabilities of the individual states based on information available up to t–1 moment in time, relevant to the t^{th} observation.

In the case of the first observation, the probability of the individual states will be as follows:

$P(S_1 = 1, S_0 = 1|F_0) = p_{11}\pi$

$P(S_1 = 1, S_0 = 2|F_0) = (1 - p_{22})(1 - \pi)$

$P(S_1 = 2, S_0 = 1|F_0) = (1 - p_{11})\pi$

$P(S_1 = 2, S_0 = 2|F_0) = p_{22}(1 - \pi)$

Generally speaking, by applying the chain rule, the probabilities of the individual states (filtered probabilities) for the t^{th} observation will be:

$P(S_t = 1, S_{t-1} = 1|F_{t-1}) = p_{11}P(S_{t-1} = 1|F_{t-1})$

$P(S_t = 1, S_{t-1} = 2|F_{t-1}) = (1 - p_{22})(1 - P(S_{t-1} = 1|F_{t-1}))$

$P(S_t = 2, S_{t-1} = 1|F_{t-1}) = (1 - p_{11})P(S_{t-1} = 1|F_{t-1})$

$P(S_t = 2, S_{t-1} = 2|F_{t-1}) = p_{22}(1 - P(S_{t-1} = 1|F_{t-1}))$

Step 2: at the t^{th} observation of the dependent variable (y_t) we update the probabilities of the individual states in the given period on the basis of new information (by applying the chain rule and the joint probability theorem):

$$P(S_t = 1, S_{t-1} = 1|F_t) = P(S_t = 1, S_{t-1} = 1|F_{t-1}, y_t) = \frac{f(S_t = 1, S_{t-1} = 1, y_t|F_{t-1})}{f(y_t|F_{t-1})}$$

$$P(S_t = 1, S_{t-1} = 2|F_t) = P(S_t = 1, S_{t-1} = 2|F_{t-1}, y_t) = \frac{f(S_t = 1, S_{t-1} = 2, y_t|F_{t-1})}{f(y_t|F_{t-1})}$$

$$P(S_t = 2, S_{t-1} = 1|F_t) = P(S_t = 2, S_{t-1} = 1|F_{t-1}, y_t) = \frac{f(S_t = 2, S_{t-1} = 1, y_t|F_{t-1})}{f(y_t|F_{t-1})}$$

$$P(S_t = 2, S_{t-1} = 2|F_t) = P(S_t = 2, S_{t-1} = 2|F_{t-1}, y_t) = \frac{f(S_t = 2, S_{t-1} = 2, y_t|F_{t-1})}{f(y_t|F_{t-1})}$$

where

$$\begin{aligned} f(y_t|F_{t-1}) = {} & f(y_t|S_t = 1, S_{t-1} = 1, F_{t-1})\, P(S_t = 1, S_{t-1} = 1|F_{t-1}) \\ & + f(y_t|S_t = 1, S_{t-1} = 2, F_{t-1})\, P(S_t = 1, S_{t-1} = 2|F_{t-1}) \\ & + f(y_t|S_t = 2, S_{t-1} = 1, F_{t-1})\, P(S_t = 2, S_{t-1} = 1|F_{t-1}) \\ & + f(y_t|S_t = 2, S_{t-1} = 2, F_{t-1})\, P(S_t = 2, S_{t-1} = 2|F_{t-1}) \end{aligned}$$

and

$$f(y_t|S_t = 1, S_{t-1} = 1, F_{t-1}) = f(y_t|S_t = 1, S_{t-1} = 2, F_{t-1}) = \frac{1}{\sqrt{2\pi\sigma_1^2}} exp\left\{-\frac{[y_t - \mu_1]^2}{2\sigma_1^2}\right\}$$

$$f(y_t|S_t = 2, S_{t-1} = 1, F_{t-1}) = f(y_t|S_t = 2, S_{t-1} = 2, F_{t-1}) = \frac{1}{\sqrt{2\pi\sigma_2^2}} exp\left\{-\frac{[y_t - \mu_2]^2}{2\sigma_2^2}\right\}$$

From this, on the basis of information available in the t^{th} moment in time, the probabilities of the individual states can be calculated (using the theorem of complete probability), which can be used in Step 1 of calculating the changes of state in the $t + 1$ moment in time:

$$P(S_t = 1|F_t) = P(S_t = 1, S_{t-1} = 1|F_t) + P(S_t = 1, S_{t-1} = 2|F_t)$$

$$P(S_t = 2|F_t) = P(S_t = 2, S_{t-1} = 1|F_t) + P(S_t = 2, S_{t-1} = 2|F_t) .$$

With these steps, the probabilities required to calculate the value of the likelihood function can be established for every moment in time; that is, with the given parameters (θ) the value of the likelihood function can be calculated.

The estimation of the parameters can be performed analytically with the quasi-maximum likelihood method, by maximising $L(\theta)$or by means of simulation techniques. In the first case, calculation of the values of the likelihood function for every new θ value is performed by means of updated probability estimation in the above-described 2-step process (*Hamilton*, 1993). In the second case, using the parameter values and information about the states, a summing of the conditional density functions is performed by means of simulation techniques, by generating possible values for the state variable (*Das–Yoo*, 2007).

The value of the forecast can be calculated on the basis of the estimated parameters. The prediction of the probability of the individual states is established by means of the above-described steps, starting out from the estimated probabilities related to the last sample element. Accordingly, the estimated value is ($\hat{y}_{t+h}$) is $\hat{\mu}_{s1_{t+h}} = \widehat{\beta_1} x_{t+h}$ and $\hat{\mu}_{s2_{t+h}} = \widehat{\beta_2} x_{t+h}$, for the two states, weighted with the state probabilities.

After the parameters have been estimated, the momentary probability of the individual states can also be estimated on the basis of the entire sample $P(S_t = 1|F_T)$, $P(S_t = 2|F_T)$ smoothed probabilities).

Due to the presence of Markovity, the model can be predicted on a rolling basis because the transition probabilities will always depend on the last state. Where a model is estimated by means of a given set of observations (with a length of t), the transition probabilities relative to $t + 1$ will be available. Using these, as well as the predicted values of the explanatory variables for the next period ($t + 1$), the value of the dependent variable can be estimated. Next, adding this estimated value to the model, we re-estimate it, which will result the transition variables relative to ($t + 2$), meaning we can go on to predict for $t + 2$, if the values of the explanatory variables are available.

4. TEST RUNS AND RESULTS

4.1. The collection and preparation of data

The above model was tested on actual macro data and Third-pillar credit risk capital requirement portfolio data where a wave of crisis was simulated corresponding to the underlying default rata. The data used included the Basel 2 Pillar III Disclosures of the following institutions: Budapest Bank (2008–2013), CIB Bank (2008–2013), Erste Bank (2008–2013), K&H Bank (2008–2013), MKB Bank (2011–2013), OTP Bank (2008–2013), Raiffeisen Bank (2008–2013) and UniCredit Bank (2008–2012). A total of 44 data points were available to us (as a limitation of public data sources) for our test run, making data enrichment necessary.

Data collection extended to bank risk databases and included the collection of macro variables as explanatory variables. In an effort to help better follow the movement of risks, relative calculations are necessary or the absolute values need to be trend-filtered. With respect to the analysis, it is better to define the relative values, because the risk values themselves are typically defined in terms of risk weight (RW) and only then will the bank calculate their absolute value (the capital requirement).

Macro-variables were collected at a monthly level during the period from January 2003 to the end of 2013. In the first round, 71 variables were established; however, because many variables were not available retrospectively for long enough, we used the following variables in the present analysis: GDP volume index and changes thereof, national reference rates for CHF and HUF and changes thereof, EUR and CHF foreign exchange rates and changes thereof, unemployment rates and changes thereof.

Unfortunately, the availability of related data does not allow for usage of the entire time series. Because GDP data are published quarterly, a 3-month moving average was calculated for GDP as a delta variable in the final monthly database.

Credit risk is the only one of the evaluated risk types (credit, operational and market risks) that helps to describe the probability of the onset of crisis and that helps to describe individual states. While this analysis sought to set up a regime-switching model for the development of both operational risk and market risk, the level of risk in these categories does not depend on crisis, at least the Markov method did not straightforwardly distinguish between crisis and normal periods. Accordingly, we continued to focus on credit risk.

In order to obtain a database that reflected the data intensity of macro-variables, it was necessary to break down (enrich) the annual data of the individual banks. Lacking actual data, expert assumptions had to be introduced into the model.

Naturally, within an institution there is no obstacle to the use of actual monthly data.

Relative annual values were extrapolated from annual bank data, for the given year and institution, with the help of the following formula:

$$Credit\ risk: R_{credit,bank} = \frac{Credit\ risk}{Total\ credit\ portfolio}$$

The relative indicator (comparing periodic data to the average) is then calculated for each year.

Formally: $RI_{r,bank} = \frac{R_{r,bank}}{\sum_{i=1}^{n} R_{r,bank}/n}$

where *r* is the given risk type, *i* is the serial number of available years, *n* the number of available years for the given bank.

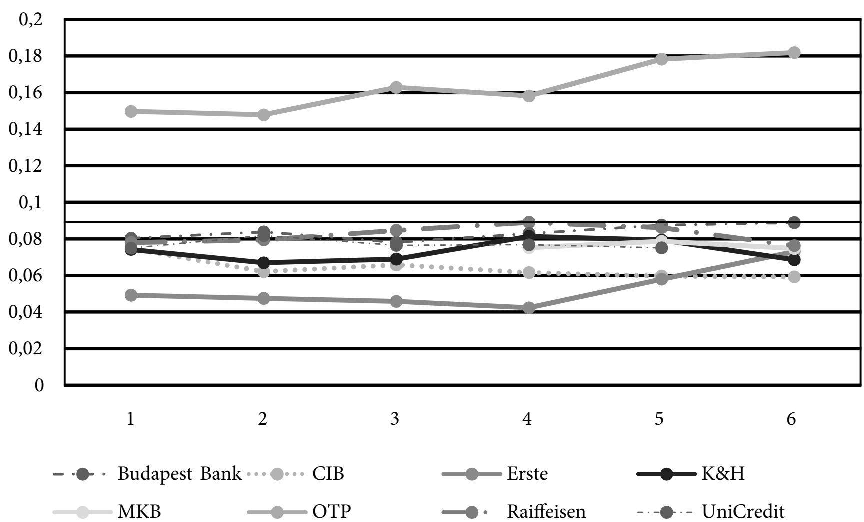
Where the above formula fails to yield a real value (e.g. R_r is missing), the value of RI_r is 1.

In the case of credit risk, the simulated crisis-based extent of credit risk was used, and weighted by multiplying the simulated value with the RI_r value for the given bank.

Simulation was necessary because the extent of credit risk was not particularly correlated with risk, so it had to be replaced with an adequately parameterised, simulated risk event. The original annual relative risk was as follows:

Figure 1

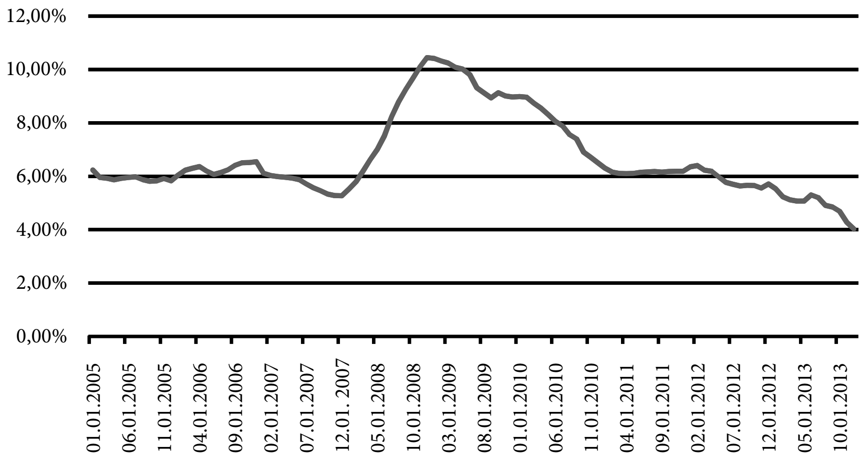
Relative credit risk of individual banks (RI_r)



The simulated time series was as follows:

Figure 2

Extent of credit risk used for data enrichment, representative of crisis



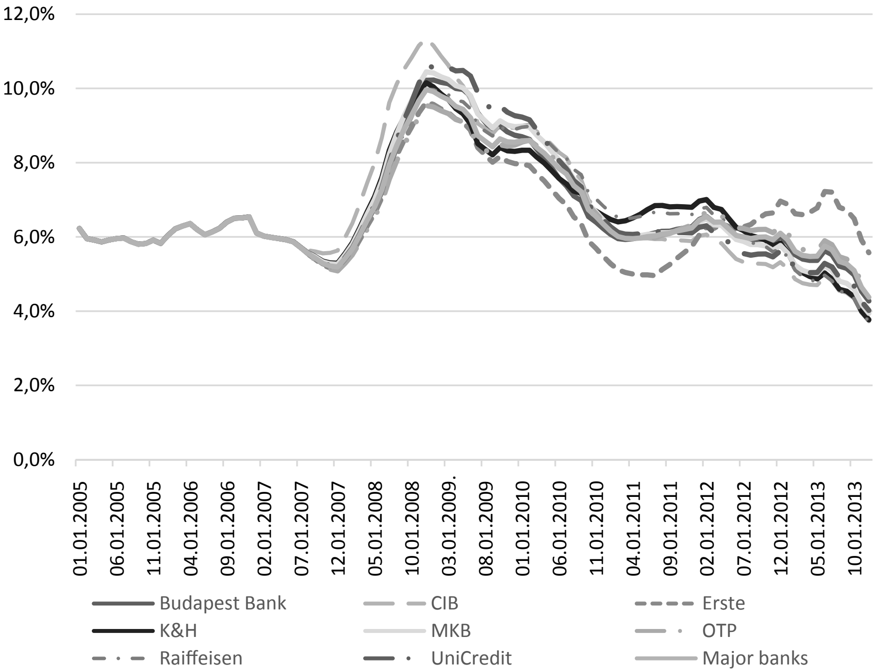
The assumed extent of credit risk spikes as a result of crisis, but returns to normal levels as the economy returns to normal.

In an effort to avoid sharp jumps, the break-down used a moving average across the individual years, calculating the annual average value of RI_r by using the current value of the given period, and looking back 6 months and 5 months ahead. That eliminates the large jumps that otherwise characterised the weighting of the data sets.

Accordingly, only the mid-month value (July) is accorded the clean annual value of RI_r and other dates carry a different weight and are determined from the data of adjacent years.

Multiplying the resulting moving average with the simulated credit risk value gives typical outcomes for the individual banks, as illustrated below.

Figure 3
Assumed complete credit risk development for individual banks

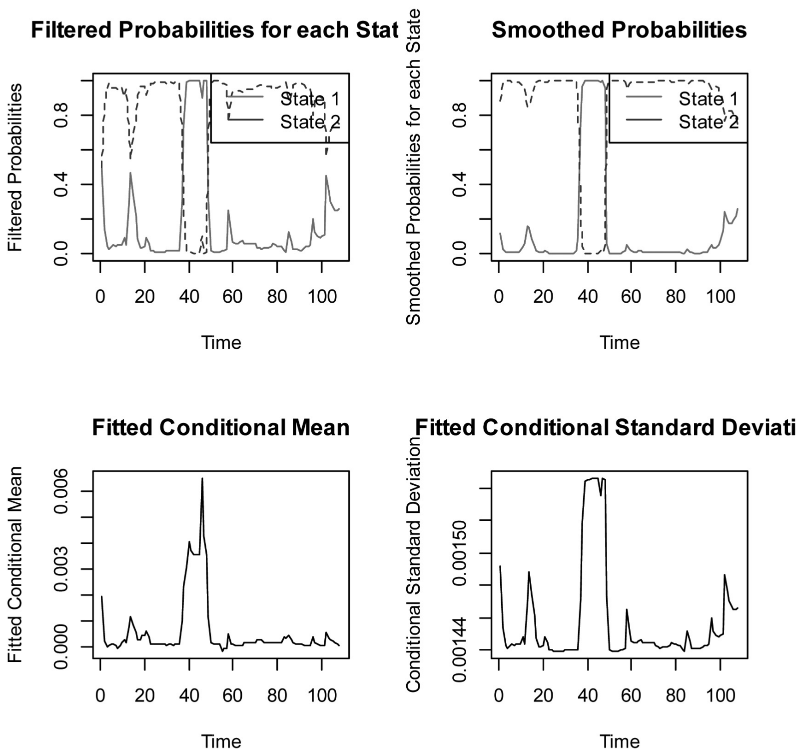


The credit risk break-down has yielded an interpretable set of data regarding the extent of credit risk development for each and every bank. Only Erste Bank somewhat sticks out. The year 2007 and the periods before that have a standard rate for each bank, due to the fact that only macro-data were available, not risk data. That is because legislation making the publication of such data mandatory has only been in force since 2008.

4.2. Model specification

The model followed the formalised specification presented in Chapter 3. The analysis yielded the following crisis-period estimations:

Figure 4
Parameterisation of the Markov regime-switching model



The above model is one example of many possible models estimating crisis. As it can be seen, the model provides the smoothed probability of two states, which can be used to estimate correlation relations and prediction.

The crucial factor among individual risk models was the extent to which they are able to predict the shifts of the crisis period, and to which the estimated state space changed as a result of changing input data.

The above models were run with a delta value, as well as with absolute GDP values, but the latter performed badly or failed altogether (e.g. the model estimated approx. 100% for every period in one state).

After expert evaluation, we chose two of the above working models for further analysis.

– **Base rate model**. The model includes the base rate and its changes. This was the only model that was able to estimate the state space in a way that for the most part it estimated the non-crisis period, and the crisis period at the time in that the individual aggregation calculations of the individual states are weighted and multiplied with the state probability values, which gives the correlation table for any given time.
– **GDP change and forint base rate change model.** The target variable is best described by this model. In the case of individual states "peace-time" is the 50–50% proportion of the two states (i.e. a kind of average), so "normal state" values are also contingent on this weighting, from which the correlation value relative to the "peace-time" period can be calculated.

4.3. Prospective risk estimation

Created in monthly increments, the above-presented forecast was made for a period of 12 months. With the help of the model we extrapolated on the basis of available data the development of risk that can be expected in the future (naturally this can be regarded as ceteris paribus development, free from extra "shock effects").

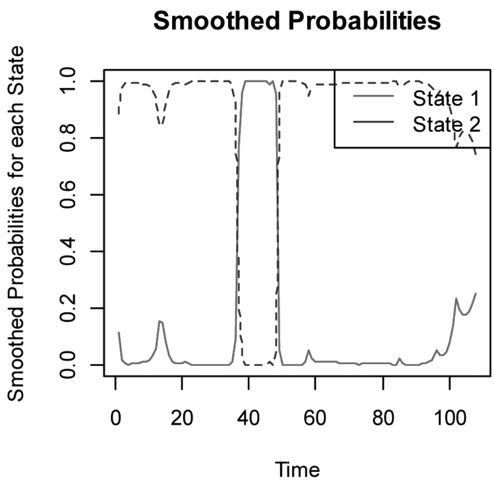
The parameterisation period lasts from early 2005 to the end of 2012. The forecasted period in our analysis was the entire year 2013. According to the model structure, portfolios are generally expected to improve and further risk reduced, assuming no structural changes occur.

4.4. Integrating risk in the Markov model

The smoothed probabilities yielded by crisis prediction will be examined to prevent individual outliers from adulterating the results of our crisis prediction process. Because the two models estimate two structurally different economic contents, we need to establish limits accordingly.

In the case of the base rate model, natural choice is an option, that is, higher-than-50% crisis status estimation predicts crisis. The model is illustrated as follows.

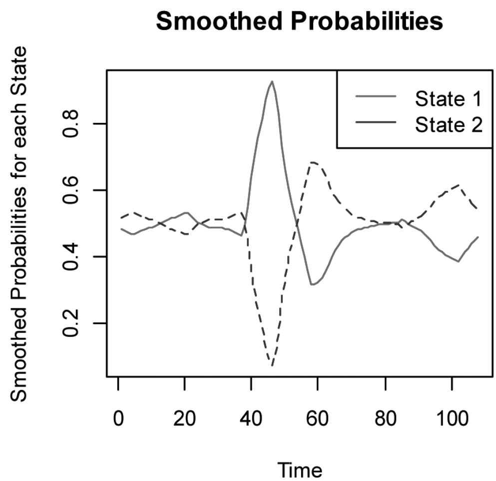
Figure 5
Parameterisation of base rate model from macro-data



The graph runs high in times of crisis, clearly indicating regime change. The estimated 50%, smoothed probability State1 is the period from 31.12.2008 to 31.12.2009.

Let us see what the other model produces and what kind of a model can be worked from GDP and interest rate change data.

Figure 6
GDP and interest rate change model on the basis of macro-variables



On the basis of the model even the 50–500% domain predicts normal periods. The use of an approx. 60% threshold will predict a crisis period (red graph) and the probability of risk increase. In this condition, risks can be expected to rise sharply, while the broken-curve indicates the reduction of risk.

The crisis period marked by the red graph and the 60% threshold also covers a 12-month period, albeit slightly shifted one. The Markov regime-switching model predicts increasing risks for the period from 30.04.2008 to 31.03.2009.

Correlations between simulated portfolios for the individual institutions can be determined, yielding the required crisis and non-crisis-period correlation matrices.

Correlation results show that crisis considerably changes the correlation structure. This means that previously stable correlation relationships will become altered, worse and probably entail higher capital requirements. The effect is different for the individual banks.

The analysis shows how, thanks to the parameterisation of the Markov regime-switching model, the correlation matrices can be determined. Accordingly, the purpose of the calculation is to weigh together the different matrices and the predicted Markov state-space probabilities, and determine the general effective correlation matrix for the following year, which in turn enables the calculation of diversification gains (vs. simple addition).

Since correlations in normal periods are typically lower, the results of the Markov regime-switching model are somewhere between the correlation model and the simple addition model.

Correlation relationships are typically stable. Using accurate bank data (not just annual data) the model can even more accurately estimate individual relationships and more precisely describe the current state of reality, determine relationships with the state of the macro-economy, and it can better predict future development of extents of risk.

One possible disadvantage of the model is that it assumes a stable loaning environment, which is not always the case and might therefore entail regime change in the regime-switching model where loaning processes induce greater change. In this case the model has to parameterised for a shorter period; however, the set of parameters must contain a crisis period in order to make the state space applicable.

5. A COMPARISON OF THE MARKOV MODEL AND THE BASIC MODEL

Few models of capital requirement aggregation exist. In conformity with disclosure legislation, the institutions use two of the basic models. Most of them use simple addition, while others use variance-covariance weighting which assumes a stable relationship between risks. On the basis of the current supervisory guidelines, the diversification effect cannot be taken into consideration in aggregating risks, so all of the institutions simply add up risks.

One of the greatest shortcomings of the variance-covariance method is that typical non-crisis-period relationships change considerably during crisis, meaning that the diversification effect resulting from the independent changes of risks can be taken into consideration to a lesser extent in times of crisis.

The authors of this paper primarily sought to develop a methodology that rejects stability and is able to give separate predictions for crisis and non-crisis periods, and, depending on the input parameters, is able to provide a conditional risk estimate.

A state-dependent system of weighting was developed, which could establish independent economic situations, and which, depending on the input macro-parameters, could determine the type of future aggregation. The historical data yielded two types of correlation whose conditional addition provides the expected future correlation matrix which is best able to aggregate the results of elementary capital calculations.

Being a model – that is, a capital model that quantifies an extreme manifestation of risk – it cannot be tested directly, and quantified statistical indices cannot be created for model errors. Acceptance in principle is required for any specific capital aggregation model.

Depending on the choice of principle, the aggregation logic is able to produce exact aggregation as well as countercyclical aggregation. We examined these two models.

5.1. Exact aggregation

According to the logic of exact aggregation, we use the actual variance-covariance matrix of the individual states. Let V_0 be the variance-covariance matrix measured in a normal period and V_c the variance-covariance matrix measured in a crisis period.

On the basis of the Markov regime-switching model, the estimated crisis state probability for period t is $S_1=1-S_{0,t}$. Let the estimation period be $T=t_1,t_2\dots t_n$, and the forecasted period $F = t_{n+1}, t_{n+2} \dots t_{n+k}$.

The prediction process, using exact aggregation, will be as follows:

$$V_F = \frac{\left(\sum_{j=n+1}^{n+k} S_{v,t}/k\right)\cdot V_v + \left(\sum_{j=n+1}^{n+k}\left(1-S_{v,t}\right)/k\right)\cdot V_0}{2}$$

That is, the accurate estimate is the arithmetic mean, weighted with state probability, of the identical elements of matrices V_0 and V_V. The method affords an accurate prediction of the correlation elements, which ranges between the crisis-time and normal correlation levels.

The methodology will yield a higher forecast where the likelihood of crisis is higher. The estimation will be accurate and the model will suit currently required capital requirements.

5.2. Countercyclical aggregation

Exact aggregation results in a procyclic approach on the capital side, which is an undesired effect in the process of determining capital requirement. It is advisable, therefore, to correct what seems to be a logical approach.

Even if one succeeds in parameterising a stable and good-quality regime-switching model, due to accurate predictions, the current exact capital requirement will cause the calculated extent of risk to fluctuate in crisis. Because it is assumed that all types of risk increase in times of crisis, correlation values too increase, the diversification effect will be lower and the resulting graphs will fluctuate too. Due to the fact that a well-parameterised macro model has an excellent power of distinction, the established risk domain can be very extreme, resulting in the forecasted average capital requirement to fluctuate considerably too.

On the whole it can be established that the only aggregation logic capable of yielding a stable extent of risk spanning many periods is one that is able to provide a similar risk level irrespective of the type of period, and where temporary changes the general risk rating of individual clients. If an institution develops a macro model and aggregation system that sensitively reacts to current changes of risk, the extent of capital requirement will, in spite of this, reflect the change of risk aggregation level, and due to the effect of estimated macro parameters, the extent of capital requirement will become unstable.

Where the institution makes an accurate calculation of the extent of capital risk, it will closely approach its actual losses. Accordingly, its capital requirement will become procyclic and make the institution vulnerable in time of crisis, since it will be expected to raise that expensive required capital in time of crisis.

Consequently, it is worth developing a countercyclical aggregation logic, which works contrary to expectation: it does not suggest raising additional capital in times of crisis, and in a normal period it does not allow for low correlation, but uses crisis-time values. This version of the aggregation model always expects contrary movement compared to current status probabilities, and assumes the reversal of expected status probabilities. In time of crisis, it bears in mind the next upturn, while in a normal period, pessimistically, it prognosticates the advent of an unexpected crisis.

Accordingly, compared to exact aggregation, the following formula gives the aggregation result. The symbols are the same as before:

$$V_F = \frac{\left(\sum_{j=n+1}^{n+k} S_{v,t}/k\right) \cdot V_0 + \left(\sum_{j=n+1}^{n+k}\left(1 - S_{v,t}\right)/k\right) \cdot V_v}{2}$$

Only the indices of the two variance-covariance matrices have changed, and accordingly, the result of the aggregation has turned and essentially become countercyclical: it provides for increased capital requirement and aggregation logic more lenient to risk; and in normal periods it does not allow consideration of a high diversification effect, preparing the bank for crisis through a slightly higher extent of capital requirement.

6. Conclusion

Simple models that use historical averages or add up capital values represent an overly conservative approach in predicting and aggregating capital. On the other hand, models that take into diversification into account are very slack, enabling the reduction of risk, which the bank can only deal with by means of external add-ons.

The model developed in this analysis affords a relatively accurate prediction of capital requirement by means of a subtle methodology. In parameterising the methodology the modeller is required to thoughtfully select the parameters predicting crisis in a way that it should actually work.

However, the result will closely approach actual capital requirement. The crisis can also be managed; crisis situations can be prepared for in advance, provided the bank employs the countercyclical model.

The methodology is, in a way, a conservative approach to the covariance method – with a considerably reasonable end result.

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