

THE DEBATED ROLE OF ERGODICITY IN (FINANCIAL) ECONOMICS

(A treatise on the applicability of the ergodic hypothesis to economics within the context of intellectual history)¹

Iván Bélyá cz

ABSTRACT

The role of the ergodic hypothesis in natural sciences

The emergence of the ergodic theory in economics – the role of *Samuelson*

The specific interpretation of ergodicity in the post-Keynesian school of thought – the example of *Davidson*

Is there a connection between *Keynes*' concept of uncertainty and the ergodic axiom?

An unfairly forgotten thinker: *Shackle*, the first adherent of the Keynesian concept of uncertainty

Uncovering the ontological foundations of uncertainty

The erroneous notion of the role of time, as an ultima ratio against the prevalence of the ergodic axiom in economics

JEL codes: G1, G10, G12, L12

Keywords: ergodicity, uncertainty, epistemology and ontology of uncertainty, nonergodic

INTRODUCTION

This paper deals with the role of ergodicity in economics, within the framework of a comprehensive analysis in the context of theoretical history. The considerable ambiguity surrounding the definition of ergodicity is reflected in the statements by Samuelson that "...a 'stable' stochastic process [...] eventually forgets its past and therefore can be expected in the far future to approach an ergodic probability distribution" (Samuelson, 1976, p. 2); and "...the connection

¹ The present scientific contribution is dedicated to the 650th anniversary of the foundation of the University of Pécs, Hungary

between ergodic processes and non-linear dynamics that characterizes present efforts in economics goes unrecognized” (Samuelson, 1976, p. 2).

The rational expectations hypothesis and the domination of the neoclassical school of thought have elicited a post-Keynesian critique. This has subsequently opened the way to a reconsideration of the old Keynesian concept of uncertainty as a fundamental and unquantifiable phenomenon. In this way, the Keynesian concept of uncertainty undermines the foundations of the rational expectations hypothesis. Neoclassical economists disregard historical time and uncertainty, ignoring the fact that economic actors – in the absence of perfect knowledge – try out a variety of approaches and solutions to decision-making problems of a non-routine nature. Classical economists believe implicitly, and neoclassical economists believe explicitly, that market participants have perfect knowledge of future events, which is equivalent to saying that they face only measurable risks.

Keynes viewed the economic system as passing through calendar time from an irrevocable past to an uncertain, statistically unpredictable future, where individuals make spending decisions in the knowledge that they are unaware of the future outcomes. All long-term decisions conceal uncertainty: all long-term decisions are vulnerable in one way or another to unexpected and unacceptable outcomes. In Keynes’ theory – in contrast to classical theory or Samuelson’s approach – people recognise that the future is uncertain.

We can differentiate two theories affecting uncertainty and decision-making, namely the classical immutable economic reality and Keynes’ transmutable economic reality. The concept of the classical, immutable economic reality states that economic actors operate in a world of perfect certainty, with full knowledge of an unchanging external economic reality that governs economic outcomes in the past, present and future. This means that economic actors already know the future, or are able to predict it, and to form rational expectations of future outcomes. Keynes’ theory of transmutable economic reality, on the other hand, denies the classical theory of an immutable economic reality, instead emphasising fundamental uncertainty as a basis in defining a transmutable economic reality. Fundamental uncertainty asserts the impossibility of forecasting future knowledge, even when actors make use of the totality of present knowledge.

We can draw the conclusion that while, on the one hand, the theory of a changeless, perfect certainty (ergodicity) supposes the availability of total knowledge for the forecasting of future outcomes, Keynes’ theory of transmutable economic reality, on the other hand, takes a contrary view, presupposing that the economy functions in an uncertain (nonergodic) world.

THE ESSENTIALS OF THE ARGUMENT

In physics, the ergodic hypothesis is a basis for calculating probabilities which relies on existing historical data for the calculation of probabilities in order to be able to prepare actuarial forecasts of future outcomes. It is best to regard the ergodic theory as part of the search for satisfactory conditions ensuring the congruence of the space and time averages. Time averages are calculated from data in series over time, i.e. from observations specific to time periods, while space averages derive from a cross-section of data containing the observations of individual realizations at given points in time. The ergodic theory played a major role in the foundations of statistical mechanics, but it is a big question whether ergodicity is absolutely necessary for the foundations of economics.

If we are searching for the path that led from the ergodic hypothesis in physics to the ergodic axiom in economics, then clarification of the role Samuelson played in this is imperative. It is an undeniable fact that Samuelson accepted the ergodic hypothesis as an indispensable condition of economics, and that by reinforcing it, pronounced it the *sine qua non* of the scientific method in economics. We must firmly regard the work of Samuelson (1965) as the first to apply the random walk hypothesis to financial markets in modern economics. The fact that Samuelson did not unravel the essential details of the application of ergodic theory to economics had far-reaching consequences. When Samuelson encouraged people to regard ergodicity as a means of reinforcing the mathematization of economics, then he essentially formulated a declaration of scientific theory. If, on the one hand, Samuelson was convinced of the undeniably random walk nature of price behaviour prevalent on financial markets, then he could not simultaneously have been an unconditional adherent of ergodic theory.

In the adaptation of ergodicity to economics, Samuelson presented progress towards equilibrium as the preeminent factor, while Davidson highlighted the predictability of economic time series. Davidson built his entire work in this area on the ergodic/nonergodic dichotomy, where the latter term can be regarded as a negation of the former. His theoretical endeavours were directed at demonstrating that economic processes in general – and the behaviour of securities in particular – are not ergodic. To disprove the ergodic axiom, Davidson needed the logical impossibility of taking a sample from the future: since such a sample is self-evidently impossible, the ergodic process must permit the analyst the assumption that samples of past and present data are equivalent to a sample from the future. The nonergodic theoretical variant constructed by Davidson was at once a denial of the ergodic axiom and a critique of the rational expectations hypothesis. Although in verbal arguments Davidson strongly criticized Samuelson's indispensability argument with respect to the ergodic axiom, in

truth – in the theoretical sense – he was opposing the theory of rational expectations. According to Davidson, decision-makers recognize that neither analysis of historical data nor present market indications can be expected to provide reliable statistical or intuitive assistance in gaining knowledge of the future.

Hicks (1979) rejected faith in the dominance of ergodic processes in economic phenomena. In his view, all economic data are linked to dates, and relationships established between sets of data exist within a given period: timeless, stable relationships are in reality only conceivable in laboratory experiments in the natural sciences. Theoreticians of mainstream neoclassical economics reject the post-Keynesian concept of ergodicity, and Davidson's ergodic/nonergodic dichotomy as a part of this.

At the focus of debate for decades is the question of what attitude Keynes might have taken to the ergodic axiom and the rational expectations hypothesis. Keynes never explicitly declared that his general theory would have demanded rejection of the ergodic axiom. Instead he was merely against the application of probability analysis in forecasting the future, while also declaring that there is no basis for developing scientific calculations supporting actuarially certain knowledge of future outcomes. In the Keynesian vein of thinking, uncertainty is not linked to probable knowledge, but precisely to its absence. According to Keynes, information is incomplete and uncertainty about the future generally makes it impossible for entrepreneurs to form rational expectations, a fact that carries definitive significance with regard to their investment decisions. Application of the ergodic axiom leads to the assumption that people can have actuarial knowledge of the future; when Keynes states that we cannot know the future, and when classical theory leads to all manner of "falsities," then it is merely a logical alternative for Keynesian economic theory to concede that existing economic data (facts) are generated by a nonergodic stochastic process. And if data are generated by a nonergodic system, then there is no scientific method permitting the future to be actuarially calculable from the existing database. The future must be uncertain, irrespective of just how much historical data has been gathered and analyzed. Keynes disputed the views of all those who believed in the existence of a general regularity. But Keynes – *expressis verbis* – did not reject the ergodic hypothesis, and did not base his concept of uncertainty on the idea of nonergodicity, and this is still true even if the Keynesian sphere of thought stood in close proximity to this theory.

Shackle constructed a formalized model, the goal of which was to simultaneously grasp the mental processes and the non-recurring, irreversible nature of economic decisions in the present. He emphasized that decision-making in situations where information has been gathered from the past relies on an inadequate basis for evaluating future outcomes, since this basis will inevitably be

of a creatively changing nature. In Shackle's view, individual choices are made between alternatives that are subjective representations of alternative future "sequels to action," not between future sequels themselves. He strongly emphasized the role of Keynesian uncertainty in investment decisions, and furthermore the unpredictability and peculiar variability of investment outcomes. According to Shackle, potential surprise and uncertainty can only arise with respect to crucial decisions, such as major capital investments, for example. Routine decisions, on the other hand, may easily be governed by ergodicity and are somewhat predictable, including much of consumer behaviour.

In an uncertain world based on the ontological concept of uncertainty, the future – prior to its unfolding – cannot be known, irrespective of the calculating capabilities attributed to individuals. Ontological uncertainty refers to indeterminism on the level of reality, as well as to the logical impossibility of knowing fundamental categories and entities of future reality. According to *Bronk*, ontological uncertainty implies the impossibility of knowing the categories and possible nature of future things which must still be created or are yet to evolve. In conditions of uncertainty, the expectations on which decisions are based depend upon imagination as well as reason; they are mediated through narratives and histories, and contain sentiments and emotions. Fundamental uncertainty posits the impossibility of predicting future knowledge, even when actors are utilizing the entirety of present knowledge. When the environment is fundamentally uncertain, then knowledge of the past does not provide sufficient information for prediction of the future, since existing knowledge or fundamentals will not be linked to the future.

Uncertainty can be characterized by unknowable future values and probabilities linked to a given location and time. If there is one thing that renders the applicability of the ergodic hypothesis to economics fundamentally questionable, it is the elimination of the role of time. Ergodicity, as a presumption necessary for the existence of equilibrium, enables the individual to formulate statements with respect to a system without having to observe every possible realization of conditions within that system. A single trajectory is sufficient to be able to deduce future behaviour in its entirety, at least on the basis of probability. All this leads to the elimination of time in the scientific description of the economic world. And yet time is a medium that surrounds and envelopes economic processes, where occurrences within these processes, and the uncertainty thereof, are linked to points or periods in time, and cannot be separated from time itself. We thus regard assumptions about the relevance of ergodicity as false because we have doubts over the timelessness and immutability of economic processes.

THE ROLE OF THE ERGODIC HYPOTHESIS IN NATURAL SCIENCES

The Encyclopaedia of Mathematics (2002) defines ergodic theory as a “metric theory of dynamical systems. The branch of the theory of dynamical systems that studies systems with an invariant measure and related problems.” From the last third of the 19th century until the end of the first third of the 20th century, numerous important scientific findings led to the aforementioned general definition of ergodic theory. Up until the middle of the 19th century, the laws of physics were grounded within the regularities of deterministic rational mechanics. *Maxwell* (1867) was one of the first to determine that probabilities can be applied to the description of the behaviour of gas systems. The Maxwell distribution can be defined based on the velocity of gas molecules, providing the probability for the relative number of molecules with velocities within a certain range. Applying a mechanical model involving molecular collisions, Maxwell demonstrated that, in thermal equilibrium, the distribution of molecular velocities is “stationary” in nature, not changing shape as a consequence of expected molecular collisions.

Boltzmann (1884) aimed to determine whether the Maxwell distribution would emerge at the limit, no matter the initial state of the gas. According to *Poitras* (2012), in order to study the dynamics of the equilibrium distribution over time, Boltzmann introduced the probability distribution of the relative time a gas molecule has a velocity within a certain range, while still retaining the notion of probability for velocities of a relative number of gas molecules. Boltzmann primarily examined problems of the kinetic theory of gases, formulating dynamic properties of the stationary Maxwell distribution – i.e. the velocity distribution of gas molecules in thermal equilibrium. From the early 1870s, Boltzmann took his own inquiries one step further to determine the evolution equation for the distribution function. The first mention of the term “ergodic” is in the aforementioned paper by Boltzmann (1884; op. cit. p. 78).² According to the ergodic hypothesis, the average behaviour of the macroscopic gas system, which can be objectively measured over time, is interchangeable with the aver

² ROSSER (2003) revealed that use of the term “ergodic” was preceded by the new word “ergode” coined by BOLTZMANN (1884). EHRENFEST – as a student of Boltzmann – proposed *ergode* as a combination of the two Greek words for “work” (*ergos*) and “path” (*hodos* – UFFINK, 2006). However, this widely accepted, conventional interpretation was challenged by GALLAVOTTI (1999), who pointed to the earlier “monode” concept formulated by Boltzmann, meaning a stationary distribution, following his efforts to formalize Maxwell’s theory more deeply, with the term “ode,” presumably derived from “*eidōs*,” denoting “similar.” He then began to employ the term “*ergomonode*,” in which the “*erg*” syllable in this case refers to energy, derived from the Greek “*ergos*” (work), so that *ergomonode* thus signifies a stationary distribution of kinetic energy.

age values calculated from the ensemble of unobservable and highly complex microscopic molecular motions at a given point in time.

Attempting to provide a mechanical foundation for thermodynamics, Boltzmann proposed a heuristic characterization of thermal equilibrium condensed to the equivalence between time averages of physical quantities on the one hand, and “ensemble averages” (space averages) of the same quantities on the other hand. For this Boltzmann needed to introduce the concept of the “statistical ensemble,” which Boltzmann referred to as the “monode” (1884, p. 79). A statistical ensemble is a notional or actual collection whose members are in principle but not in practice distinguishable, and such that a realization of the object at hand can be taken to be a randomly selected element of the ensemble. When the ensemble consists of discrete values and is finite, then as a rule identical probability is assumed for all members of the ensemble in the spirit of the “principle of indifference.”³ *Alvarez and Ehnts* (2012) highlight Boltzmann’s “brilliant intuition” that a single instance of the mechanical system is able, during its own evolution, to sample the entire collection of available microscopic states compatible with the macroscopic constraints (the ergode), and that the time spent in each state would define a natural probability distribution with respect to the ensemble. In this way, ensemble (space) averages would equal time averages.

The latter is what *Ehrenfest* and *Ehrenfest* termed the “ergodic hypothesis,” the basis for the calculation of probabilities using existing historical data to prepare actuarial forecasts of future outcomes. In formulating the ergodic hypothesis, the Ehrenfests (1912) focused on a formulation by Boltzmann whereby, given a set of underlying microcanonical trajectories, each with a mass subject to an arbitrary – time-independent – potential, all of this a phase point or ensemble, that for any given region the probability that the phase point will be in that region will be a stationary distribution. In this regard, the original ergodic hypothesis was deeply and strongly connected to the findings of probability theory in the 1930s. This form of the ergodic hypothesis proposed in effect that trajectories would return to any given zone, this being the idea of recurrence – a theorem earlier emphasized by *Poincaré* (1893) – and space-filling or measure preservation (*Rosser*, 2003).

The definition of ergodic theory cited at the start of this chapter reveals one of the culminations of the thought process represented by *von Neumann* (1932) and *Birkhoff* (1931). *Alvarez and Ehnts* (2012) emphasize that the theorems of von Neumann and Birkhoff assume as a hypothesis that a dynamical system

3 LAPLACE (1814) made this the essence of the theory of chance. Laplace’s method is not valid in the case of infinite or continuous ensembles, where some variant of “indifferent” probability distributions must be assumed.

has a time-invariant measure (which can be assimilated to the probability distribution and defines the ensemble average). All that these theorems prove, in this case, is that Boltzmann's time averages do exist, and coincide with the ensemble (space) averages. In terms of applications, at the same time, the existence (and uniformity) of an invariant measure can be regarded as proven. *Ergodic theorems are best seen as part of the search for sufficient conditions ensuring the equality of ensemble (space) and time averages.*

While Maxwell, Boltzmann and the Ehrenfests favoured the frequency approach, von Neumann (1932) formalized a more precise version. After working on stochastic problems in quantum theory, he came to study how measure theory relates to probability theory when the latter is central (Halmos, 1958). Von Neumann focused more on measure preservation, whereas Birkhoff showed that not only is measure preserved, but that a properly defined stationary ergodic system exhibits metric indecomposability such that not only is the space properly filled, but that it is impossible to break the system into two that will also fully fill the space and preserve measure.

The contributions of von Neumann (1932) and Birkhoff (1931) marked the solution to a problem in *statistical mechanics* and thermodynamics that was recognised 60 years earlier when Boltzmann laid the foundations for the hypothesis formulated by Ehrenfest permitting the theoretical phase space average to be interchanged with the measurable time average. The thermodynamic model introduced by Boltzmann was suited to explaining the dynamic properties of the Maxwell distribution, and may serve as a useful starting point for expanding the implications of ergodicity in financial economics. It is undeniable that von Neumann (1932) and Birkhoff (1931) correctly specified ergodicity, as an essential analytical tool that was not available to Boltzmann. When von Neumann and Birkhoff generalized the findings of ergodic theory in the early 1930s, the significance of the physical intuition of the kinetic gas model had already receded. Using Boltzmann's theorem as a starting point, the large number of mechanical and complex molecular collisions could correspond to the large number of microscopic, atomistic liquidity providers and traders interacting to determine the macroscopic financial market price. In this context, as Poitras (2012) points out, it is variables such as the asset price or interest rate or exchange rate, or some combination thereof, that are being measured over time, and ergodicity would be associated with the properties of the transition density generating the macroscopic variables. However, there are two fundamental difficulties associated with the ergodic hypothesis in Boltzmann's statistical mechanics – reversibility and recurrence – that are the source of problems in adaptation of the theorem to financial economics.

The aforementioned leaves no doubt that the evolution of the ergodic theory took place through several channels, largely inspired by the problems of physics, associated at various points with problems arising in various branches of mathematics, and playing a particularly profound underlying role in probability theory. We must note that this process has brought about a variety of definitions and divergent conceptual frameworks at different times with respect to what is actually meant by “ergodicity.”

The central aspect of ergodic theory is the behaviour of dynamical systems, where hypotheses can be made about their operation in the long term. This is expressed in the ergodic theory, which states that under certain conditions the time average of a function – along the full length of the trajectory – exists almost everywhere and is connected to the space average. Birkhoff (1931) and von Neumann (1932) started from the general assumption that time and space averages may diverge. If the transformation is ergodic, however, and the measure is invariant, then the time average is equal to the space average almost everywhere. This is the ergodic theorem in abstract form as formulated by Birkhoff. In a dynamical system, a transformation maps a space into itself. The set of points we get from applying the transformation repeatedly to a given point is called its trajectory or orbit. Some dynamical systems are “measure preserving,” which means that the measure of a set is always the same as the measure of the set of points which map to it.

All stochastic processes generate time series data (realizations) which can be used to calculate averages. These averages (e.g. the mean, standard deviation, etc.) form the basis for empirical knowledge about our past and current real world relationships. Time averages refer to averages calculated from a single realization over any period of calendar time. Space averages, on the other hand, are statistical averages formed over a universe of realizations observed at a fixed point of time (i.e. estimates from cross-sectional data).

Davidson (1988, p. 331) noted that we are examining ergodic stochastic processes if: a) for infinite realizations, the time average and space average coincide; or b) for finite realizations, as the number of observations increase, the time and space averages tend to converge. By time average, therefore, we mean an observed trajectory (realization) of the process, while the space average is the average of every possible state of the system.

In the assessment of *Kirstein* (2015): “Ergodicity creates a relation between the behaviour through time and the behaviour in all possible states of the system. The set of all possible states of a system builds the phase space. The state of a system (at an arbitrary point in time) is characterized by its location in phase space. If one thinks of all other possible realizations at a given point in time, one can think of them as parallel worlds. Simplified, ergodicity relates time to the

phase space.” Halmos (1949, p. 1017) concisely expressed the common essence of various approaches to ergodicity when he wrote: “The ergodic theorem is a statement about a space, a function, and a transformation.” In the mathematical sense, ergodicity or “metric transitivity” is a property of measure-preserving transformations “irreducible” to its components.

The ergodic theory played a major role in the foundations of statistical mechanics, but it is a big question whether ergodicity is absolutely necessary in the foundations of (financial) economics. Although statistical mechanics has been successfully applied to physical systems, economic systems are more complex than their physical counterparts. The root of the problem lies in the fact that “macroscopic” financial variables such as share prices, exchange rates or interest rates could only be modelled in the hope of gaining precise forecasts if a theory were to exist to authentically describe the “microscopic” behaviour of individuals and companies.

The three decades before and three decades after the turn of the 20th century produced scientific findings that laid the foundations for the applicability of ergodic theory to numerous branches of natural science – in physics, chemistry and biology, as well as in engineering science. Retracing the routes by which formulated scientific theorems were passed on, an intellectual chain of thought can be followed that clearly reveals the path along which an idea was subsequently developed. The table below shows two such intellectual routes: one traces the process of laying the ergodic foundations of kinetic theory, while the other portrays the evolution and application of abstract ergodic theory. In both chains we find master-student links that reinforce the relevance of directly transmitted intellectual influence and ideas.

Table 1:
Intellectual routes in the evolution and development of ergodic theory

Physics-based ergodic theory		Abstract ergodic theory	
Clausius (1857) Maxwell (1867)	} Microscopic mechanical model	Poincaré (1893)	Recurrence theorem
Boltzmann (1884)		Gibbs (1902)	Statistical mechanics Economic theorems on a mathematical basis
Ehrenfest (1912)	Ergodic hypothesis	Wilson	
Tinbergen	Natural law in development of time series	Birkhoff (1931), Neumann (1932)	} Abstract ergodic theory
		Samuelson (1965)	

Source: own arrangement based on *Kirstein* (2015), *Rosser* (2007), *Poitras* (2012)

The *intellectual routes* – besides many other points of interest not detailed here – draw attention to the master-student connections. Boltzmann’s former pupil Ehrenfest – in his doctoral thesis on physics – was the first to state that his ideas might also be applied for forecasting purposes in economics. From Rosser (2007), we know that in the early 1920s Ehrenfest became Tinbergen’s most important teacher. Tinbergen advocated that there exists a stable “natural law” underpinning economic time series. In his opinion, this natural law induces ergodic, stationary and homogeneous time series.

If we look for the route that led from the ergodic hypothesis in physics to the ergodic axiom in economics, then the clarification of Samuelson’s role in this is inescapable. It is a commonly held belief in the economic literature that the works of Samuelson written in the second half of the 1960s (Samuelson 1965; 1968; 1969) are the first to make mention of the applicability of ergodic theory to economics. On the other hand, it is a fact that in the subtitle to his work *Foundations of Economic Analysis*, Samuelson (1947) quotes J.W. Gibbs’ famous words: “Mathematics is a language.” As a physicist and one of the founders of statistical mechanics, Gibbs played a role in the later mathematization of economics. The influence was indirect, as during Samuelson’s years as a PhD student at Harvard his teacher was E.B. Wilson, and it is also known that Wilson was a student of Gibbs. The link between Gibbs and Samuelson, which came about through Wilson, can thus be clearly seen. In a later paper, Samuelson (1998, p. 1376) himself acknowledged that his teachers made him understand at

an early stage that economics and physics could share the same “formal mathematical theorems.” In this way, Samuelson became (perhaps) Wilson’s only disciple, and thus a “grandson” to Gibbs.⁴

THE EMERGENCE OF THE ERGODIC THEORY IN ECONOMICS – THE ROLE OF SAMUELSON

“The concept of ergodicity in economics seems to have the qualities of a shibboleth – a word or saying used by adherents of a party, sect, or belief, and usually regarded by others as empty of real meaning.”

(Alvarez-Ehnts, 2012, p.1)

(Financial) economics has always been exposed to the influence of physical science. *Regnault* (1862) was one of the first authors to bring physics closer to finance. He wrote the first statistical representation of the price fluctuations of financial assets, taking the Gaussian framework as its basis. In the 20th century, numerous concepts from physics played a role in the development of modern market theory. The best-known example of the application of physics to finance is the adaptation of the heat diffusion formula (*Bachelier*, *Black* and *Scholes*), while numerous authors have referred – implicitly or explicitly – to Brownian motion, another concept originating in the field of physics. *Jovanovic* (2001) revealed that Regnault had an unequivocal influence on the work of *Bachelier* (1900), whose writing around the turn of the century employed the Gaussian framework and Brownian motion alike as fundamental ideas in the evaluation of asset price fluctuations.⁵

The Gaussian-type description of financial reality was crystallized and reinforced when Samuelson (1965) introduced geometric Brownian motion to the description of continuity of financial time series (trajectories). Samuelson (1965, p. 43) uses the term “ergodic” without saying anything of merit beyond its mention. We must agree with the opinion of Alvarez and Ehnts (2012) that the concept of ergodicity is not a prominent element of Samuelson’s article. In his follow-ups, Samuelson (1968; 1969) repeated the old argument, adopted by Tinbergen from Ehrenfest, that natural science is dependent on the assumption

4 WEINTRAUB (1991, p. 61) reconstructed how the work of SAMUELSON (1947) can be seen as the continuation of the GIBBS–WILSON tradition in other scientific disciplines. For Samuelson, this tradition proved suitable for making economics “scientific” by presenting the essential propositions of the subject in a mathematical, easily analyzable form.

5 Similarly, empirical works published from the 1930s onwards (COWLES, 1933; WORKING, 1934; KENDALL, 1953) applied a Gaussian-type framework.

of the ergodic hypothesis. Samuelson (1968, p. 11) provides an explanation of the role played by the ergodic hypothesis in the mathematization of economics:

“.....there was an even more interesting third assumption implicit and explicit in the classical mind. It was a belief in unique long-run equilibrium independent of initial conditions. I shall call it the ergodic hypothesis by analogy to the use of this term in statistical mechanics.”

It can be seen that Samuelson (1968, pp. 11–12) writes about the ergodic hypothesis in the context of an examination of the “classical mind”: the belief in unique, *long-run equilibrium* independent of initial conditions. It is an undeniable fact that Samuelson (1969, p. 184) accepted the ergodic hypothesis as an indispensable condition of economics, and by generalizing it, pronounced it the sine qua non of the scientific method in economics. He indicated that he used the term ergodic “by analogy to the use of this term in [19th century] statistical mechanics,” in order to remove economics from the “realm of genuine history” and keep it in the “realm of science.” (Samuelson, 1969, p. 184)

According to *Dunn* (2012, p. 434), the central thrust of the argument of the post-Keynesians, eminent among them Davidson (1982–83; 1991; 1994; 1996), was to verify the ergodic hypothesis as the foundation of neoclassical economics. It is true that Samuelson (1968, pp. 11–12) asserted that the classical mind believed in unique long-run equilibrium independent of initial conditions.⁶ But Samuelson did not declare his own belief in the ergodic hypothesis, attributing this to the classical thinkers as an implicit presupposition. Accordingly, it is inaccurate to state that Samuelson, in keeping with his endeavours to create a solid scientific basis for economics, would have judged acceptance of ergodic theory as a necessary prerequisite for modern economists.

It is widely stated in the literature on finance (and economics) that a seminal article by Samuelson (1965) introduced the concept of ergodicity, with his contribution concisely expressed in the title of the article: “*Proof that Properly Anticipated Prices Fluctuate Randomly.*” This essentially means that in an informationally efficient market, prices must be unforecastable if they are properly anticipated, i.e. if they fully incorporate the expectations and information of all market participants. Samuelson assumes stationarity, but only in order to deduce that the volatility of futures prices decreases as the time to maturity decreases, and not with respect to the price of the underlying asset. Samuelson (1965, p. 48) expresses doubts regarding the contents of his article:

6 SAMUELSON (1968) wrote that when he was an immature young man his thoughts followed the classical view, accepting that the ergodic reality of the economic system recalled natural law, as TINBERGEN asserted. After reading KEYNES’s work (1936) in 1937, which represented a turning point for him, he ceased to believe this at all (ALVAREZ–EHNTS, 2012).

“One should not read too much into the established theorem. It does not prove that actual competitive markets work well. It does not say that speculation is a good thing or that randomness of price changes would be a good thing. It does not prove that anyone who makes money in speculation is ipso facto deserving of the gain or even that he has accomplished something good for society or for anyone but himself.” (op. cit. p. 48)

Samuelson is similarly cautious with respect to the fixed probability distribution of futures prices, a fundamental assumption of his theory:

“I have not here discussed where the basic probability distributions are supposed to come from. In whose minds are they ex ante? Is there any ex post validation of them? Are they supposed to belong to the market as a whole? And what does that mean? Are they supposed to belong to the ‘representative individual,’ and who is he? Are they some defensible or necessitous compromise of divergent expectation patterns? Do price quotations somehow produce a Pareto-optimal configuration of ex ante subjective probabilities? This paper has not attempted to pronounce on these interesting questions.” (op. cit. pp. 48–49)

From all this, Samuelson (1965, p. 44) came to the conclusion that “there is no way of making an expected profit by extrapolating past changes in the futures price, by chart or any other esoteric devices of magic or mathematics.” As Samuelson puts it (1965, p. 41): “In competitive markets there is a buyer for every seller. If one could be sure that a price will rise, it would have already risen. Arguments like this are used to deduce that competitive prices must display price changes [...] that perform a random walk with no predictable bias.”

In the 1960s, two important factors influenced Samuelson with regard to the analysis of price behaviour on securities markets. One was *the role of ergodic theory in economics*, and the other the assumption of a definitive role for *random walk* and *unforecastability*. We have discussed the former in detail in the preceding passages, and will give our opinion of the latter in what follows. In his essay on speculation, Bachelier (1900) described a method for modelling the future evolution of stock market prices. In his introduction, Bachelier states that “past, present and even discounted future events are reflected in market price, but often show no apparent relation to price changes.” This recognition of the informational efficiency of the market prompts him to follow this train of thought in his introduction by adding that “while the market does not foresee fluctuations, it considers which of them are more or less probable, and this probability can be evaluated mathematically.” (op. cit. p. 3)

It is regrettable that Bachelier’s valuable contribution was overlooked for such a long time, right up until his model aroused the interest of Samuelson, who began

to disseminate his work's findings from the late 1950s.⁷ Samuelson's work (1965) must be explicitly regarded as one of the first applications of the random walk hypothesis to financial markets in modern economics. *The regularity of random walk meant that no professional investor could "beat the market."* The unforecastability of changes in securities prices meant that information derived from past share prices – serial correlations, linear dependence or more complicated patterns of data – was not utilizable in the prediction of price fluctuations. It follows from random walk that any probability distribution based on past data cannot be authentically used to indicate the probability distribution that directs some future outcome. In other words, even if we knew that the future probability distribution has a lesser variance than that of the calculated probability distribution, then the past distribution would still not provide an authentic guide to future statistical averages or other circumstances around the average.

From Samuelson's observations of random walk, it can only follow that although price series may contain statistical dependences, it is not possible to deduce the direction and intensity of future changes from past changes. In the words of Samuelson (1965), share prices "fluctuate randomly," and this is also true in reality. At this point, however, the "ergodic hypothesis" and the "random walk hypothesis" conflict as opposites. For Samuelson, acceptance of ergodic theory also meant admitting the concept of stationarity.⁸ The notion of stationarity means that the probability structure of the pricing process is independent of historical time. Accordingly, the historic point in time – when the experiment is carried out – cannot in itself influence the outcome; Wald (1965, p. 52) stresses that if a process is ergodic, calculation of the time average may provide an estimate of space averages, and/or calculation of the space averages (based on cross-sectional data) may provide an estimate of time averages.⁹

7 An English version of the work also appeared thanks to COOTNER (1964). SAMUELSON wrote an (unpublished) article on these ideas in the mid-1950s entitled "*Brownian Motion in the Stock Market.*"

8 In probability theory, a stationary ergodic process is a stochastic phenomenon which exhibits both stationarity and ergodicity at once. In essence this implies that the random process is a statistical process that will not change its statistical properties with time, and that these statistical properties (such as the theoretical mean and variance) can be deduced from a sufficiently long sample of the process. Stationarity is the property of a random process which guarantees that its statistical moments will not change over time. A stationary process is one whose probability distribution is the same at all times. The ergodic hypothesis is often applied in statistical analysis. An analyst would assume that the average of a process parameter and the average of the statistical ensemble over time are the same. The analyst's assumption, rightly or wrongly, is that it is just as correct to observe a process over a long time as it is to sample many independent realizations of the same process.

9 WALD (1965, pp. 53–54) stated that "statistical averages are also known as space averages, which refer to a fixed point in time, and are formed as the averages of the universe of realizations. [...] time averages are likewise known as phase averages, referring to a fixed realization, formed as the averages of an undefined timespace."

It was on this basis that Samuelson was able to say that we must accept the ergodic hypothesis because if a system is nonergodic then it cannot be treated scientifically, meaning that a system is very insensitive to initial conditions or disturbances and to details of its dynamics.

The fact that Samuelson did not unravel the essential details of the application of ergodic theory to economics had far-reaching consequences. When quoting the seminal words (“Mathematics is a language”) of J.W. Gibbs (1902) as the subtitle to his work *Foundations of Economic Analysis* (1947), or in his equally famous work *Economics* (1948) and later essays written at the end of the 1960s (1968; 1969), *Samuelson encouraged people to regard ergodicity as a means of reinforcing the mathematization of economics, thereby essentially formulating a declaration of scientific theory. If, on the one hand, Samuelson was convinced of the undeniably random walk nature of price behaviour prevalent on financial markets, then he could not simultaneously have been an unconditional adherent of ergodic theory.*

It was on this contradiction in Samuelson’s thinking that Davidson (1982–83; 1991; 1994; 1996) built a concept of ergodicity that rested on his own interpretation. Despite the fact that neither Samuelson nor anyone else had formulated a positive concept of ergodicity applicable to economics that was analogous to the ergodic theory of natural sciences, Davidson still presumed ergodicity as an important axiom of neoclassical economics. To construct his own ergodic/non-ergodic argument, Davidson surely needed to adopt a firm stance in opposition to Samuelson’s contradictory viewpoint. Rosser (2003) writes that – according to widely repeated verbal arguments – it was *Weintraub* who drew Davidson’s attention to the contradiction in Samuelson’s viewpoint and its potential exploitation. Concerning the views of Samuelson that had crystallized by the end of the 1960s, we can state that for him, *ergodic theory meant more the striving of markets towards equilibrium, rather than the forecastability of price behaviour on financial markets.* As we shall see in the following, Davidson and the post-Keynesians rendered forecastability absolute, using it as the basis for their criticism of Samuelson and the concept of ergodicity.

The specific interpretation of ergodicity in the post-keynesian school of thought – the example of Davidson

The ergodic hypothesis – or in common parlance the ergodic axiom – was introduced to neoclassical economics by Samuelson (1965; 1968; 1969), basically by mentioning the concept without expounding on its essential details. Later Davidson (1991; 1996; 2007) placed the concept of ergodicity at the centre of his own work, primarily in order to emphasize methodological differences. David-

son placed reliance on the “ergodic axiom” in neoclassical economics at the focus of his thinking. According to Dunn (2012, p. 434), Davidson took Samuelson’s suggestion as the basis for the assumption that economic knowledge about the future rests on the axiom of ergodicity. In Davidson’s understanding (1996, p. 65), Samuelson (1968) wrote that, in searching for a solid scientific foundation for economics, modern economists must believe in a unique, long-run equilibrium independent of initial conditions. Alvarez and Ehnts (2012), however, warn that Samuelson (1968) was not asserting his own belief in the ergodic hypothesis, but merely attributing it to the classical thinkers as an implicit presumption. It is therefore incorrect to say that Samuelson, in striving to provide a hard scientific basis for the economics discipline, required that modern economists must accept the ergodic theorem.

As we saw in the first chapter, the ergodic hypothesis had a major role to play in the foundations of statistical mechanics. At the same time, when Samuelson formulated his declaration of scientific theory, it remained very much open to debate whether ergodicity was essential for the scientific foundations of economics, in view of the fact that *economic systems are more complex than the physical systems to which statistical mechanics could be successfully applied*. When the ergodic hypothesis was declared a scientific fundamental of neoclassical economics, then – without going into the essential details of ergodicity – it would have been hard to decide which part of the argument of ergodic theory could be applied to economics. Perhaps the convergence and congruence of the time and space averages, or perhaps the stationarity and homogeneity of economic time series, or the creation of equilibrium independent of initial conditions, or the observation that the ergodic hypothesis is a special case of the law of large numbers. We can find no trace in the specialist literature of Samuelson having ever ascribed significance to the ergodic theory of Birkhoff (1931) and von Neumann (1932). Strictly with respect to the natural sciences, this latter assumed as its hypothesis that a dynamical system has a time-invariant measure (which can be assimilated to the ensemble/space probability distribution and defines the ensemble average). All this supports the tendency of the time and space averages to converge. On this basis, Davidson (2006, p. 32) wrote the following: “The ergodic axiom therefore assures that the outcome associated with any future date can be reliably predicted by a statistical analysis of already existing data. The future is therefore never uncertain.” In the adaptation of ergodicity to economics – in the light of what we have discussed so far – Samuelson highlighted the progress towards equilibrium as the paramount factor, and Davidson the predictability of economic time series.

Davidson built his entire work in this area on the ergodic/nonergodic dichotomy, where the latter term can be regarded as a negation of the former. His theoretical

endeavours were directed at demonstrating that economic processes in general – and the behaviour of securities in particular – are not ergodic. A consistently recurring motif in quotes from Davidson's works is that economic processes do not obey the criteria of an ergodic system. The first citation here adapts the best-known partial theorem of statistical mechanics-based ergodic theory to the evolution of economic data series over time. According to a technical definition of ergodicity:

“Time statistics refer to statistics calculated from a single realization over any period of calendar time. Space statistics are statistics formed over a universe of realizations at a fixed point of time. [...] If the stochastic process is ergodic, then for infinite realizations, the space and time statistics will coincide. For finite realizations, the calculated space and time statistics may differ [...] but they will tend to converge [...] as the number of observations increases.” (Davidson, 1996, p. 480; 1988, pp. 331–332; 1994, pp. 89–90; 2006, p. 185)

These thoughts are followed by a surprising interpretation, which signifies a change of direction in ideas:

“If, and only if, the process is ergodic, then space or time statistics calculated from past or current market data are reliable estimates of the immutable objective probability distributions that govern any future outcome at any specific future date. Consequently, past data can be treated as if they were a sample drawn from the future.” (Davidson, 1996, p. 480; 1988, pp. 331–332; 1994, pp. 89–90)

Again and again Davidson returns in his work to the absurd assumption that “the future is a reflection of the past,” speaking of “a sample drawn from the future” and the theme of “predetermined” processes.

“...whenever analysts invoke the ergodic axiom, they are proclaiming that statistical samples drawn from past or current market data are equivalent to drawing samples from future market data. In other words [...] the future is merely the statistical shadow of the past. [...] Only if this axiom is accepted as a universal truth will calculating probability distributions (risks) on the basis of historical market data be statistically equivalent to drawing [...] samples from the future.” (Davidson, 2009, p. 328; 1996, pp. 479–480)

The assumption of predetermination is illustrated by the following quote:

“This ergodic axiom assumes the economic future is already predetermined. [...] [It] presumes that the economic future is governed by an already existing, unchanging ergodic stochastic process. Consequently, a sample drawn from the past is equivalent to a sample drawn from the

future. [...] In an ergodic world, all future events are already predetermined.” (Davidson, 2012, pp. 2–4)

On this basis, if we assume the economy to be a stochastic process, then the future outcome of any current decision is determined by a probability distribution. Logically, in order to be able to prepare a statistically reliable forecast of future economic events, the decision-maker must draw a sample from the future for analysis. *In order to discredit the ergodic axiom, Davidson needed a logical impossibility in the sampling process: given that drawing a sample from the future is self-evidently impossible, the ergodic process must permit the analyst to assume that samples drawn from past and present data are equivalent to a sample drawn from the future.* In other words, the ergodic axiom implies that the outcome at any future date is the statistical shadow of past and present market data. According to ergodic theory, probability distributions based on past data supply statistically reliable information about future events.

Accordingly, proponents of the ergodic axiom assume that *the economic future is predetermined*, given that economic processes are governed by an existing ergodic stochastic process. It is enough for decision-makers to calculate probability distributions regarding future prices and output to be able to gain reliable statistical information about the future. Once self-interested decision-makers have this reliable information about the future at their disposal, then – through their actions on free markets – they can optimally allocate resources into activities which promise the highest possible future returns.

Davidson (1988, p. 331) strives to root the fundamental ergodic principle adapted to economics in the category of statistical mechanics. He takes the view that space and time averages calculated from past realizations collapse onto the objective probability distribution that describes all possible (past, present and future) realizations, and that these time and space averages govern, and form reliable estimates of, both current and future events. The past thus sets a pattern for the future; or – to put it another way – the past reveals the future. If, however, the concept of ergodicity is generally understood to relate to stochastic processes, in a wider sense this implies – as Davidson (1996, pp. 480–481) points out – “the presumption of a programmed system, where the past, present and future reality are predetermined, whether the system is stochastic or not.”

The above barely leaves room for doubt that *there is essentially no connection between the ergodic hypothesis expounded by Davidson and the ergodic axiom raised to the canonical level (but never expounded upon) by Samuelson.* We have to make do with the assumption that the nonergodic version of the theory constructed by Davidson was at once a denial of the ergodic axiom and a critique of the rational expectations hypothesis. According to the rational expectations hypothesis, external reality takes the form of a related probability. In keeping

with this, the individual's knowledge is manifested as a subjective probability distribution, while the underlying reality, which is the object of study and prediction, can be described and comprehended as an objective or "true" probability distribution. Lucas (1981, p. 223) described the concept as "identifying agents' subjective probabilities with observed frequencies of the events to be forecast, or with 'true' probabilities, calling the assumed coincidence of subjective and 'true' probabilities rational expectations." Accordingly, it appears that "true" probabilities refer to events that are happening in the world now; events have probabilities that agents may experience, and as such, may know (or may not know).

Market participants whose subjective probabilities do not converge with the objective probabilities commit persistent and systematic errors which are manifested in loss of income. For this reason, Davidson (1991, p. 132) observes that "for the rational expectations hypothesis to provide a theory of expectational formation without persistent errors, not only must the subjective and objective distribution functions be equal at any given point of time, but these functions must be derived from what are called ergodic stochastic processes. By definition, an ergodic stochastic process simply means that averages calculated from past observations cannot be persistently different from the time average of future outcomes."

Based on the principle of ergodicity, future outcomes are known, and the probability of every outcome can be determined. Future outcomes are predetermined by present fundamentals, and given that the parameters of these do not change in time, the outcomes cannot be changed by human action. Accordingly, future outcomes can be understood merely as statistical shadows of the past. Dunn (2001, p. 573) drew particular attention to the fact that "time averages are calculated from time-series data, that is, observations that relate to a period of calendar time, while space averages are computed from cross-sectional data, that is, observations that relate to a given point in time across realizations." We must recall that the ergodic principle presupposes that a relationship of economic equilibrium must exist which is determined by a number of genuine factors that are time and path-independent in the long term. *It is at this point that the ergodic hypothesis, conceived in the context of natural science, conflicts with the attempt to adapt this hypothesis to economics. Here we must face the fact that there are limits to the extent to which the ergodic theory, as based on statistical mechanics, can be reconciled with economics.* If economic reality is ergodic, then no practice can be followed that would alter probable future outcomes, and people have no freedom to be able to change their own long-term economic futures. The ergodic principle and the objective probability environment assume not only that probability distributions have existed with respect to past

phenomena, but also that the same probabilities that determined past outcomes will govern future events as well. This means that decision-makers believe in a statistically reliable past, and that it provides an undistorted guide to the future. With regard to the presumed adaptability of the ergodic hypothesis to economics, Davidson found that the evolution of economic time series and the forecastability of future outcomes are irreconcilable with the conditions of ergodic theory. Although in verbal arguments Davidson strongly criticized Samuelson's indispensability argument with respect to the ergodic axiom, in truth – in the theoretical sense – he was opposing the theory of rational expectations. As we shall see in the following, the important thing for Davidson was to deny the adaptability of the ergodic hypothesis to economics, and as the basis for this he constructed his nonergodic framework as a theoretically well-founded theory of uncertainty. Davidson (1988, p. 331) declared at an early stage that fundamental uncertainty must be defined in terms of the arguments of ergodic theory. *We can apprehend the essence of Davidson's contribution by saying that he placed his own interpretation of uncertainty within the context of the ergodic/nonergodic framework.*

In the late 1970s and 1980s, the post-Keynesians in general – and Carabelli (1988), Fitzgibbons (1988), O'Donnell (1989) and Davidson in particular – turned against the theorems of neoclassical economics related to rational expectations. Their fundamental assumption was that the processes of financial and economic systems are nonergodic. By contrast, neoclassical economists disregard historical time and uncertainty, deliberately ignoring the fact that economic agents may have different ways of approaching and solving problems of non-routine investment decision-making, since their heterogeneous expectations – in the absence of perfect knowledge – lead them to reckon on contingencies that differ from those suggested by models of ergodic, rational expectations.

Davidson (1978) – and the post-Keynesians – follow a train of thought in which understanding of the real world under conditions of uncertainty is not based on probability distributions. This is to say that, in their view, numerous situations may arise in reality where we must confront “true” uncertainty with respect to the future consequences of today's choices. *In these instances, decision-makers in the present recognize that neither analysis of historical data nor present market indications can be expected to provide reliable statistical or intuitive assistance in gaining knowledge of the future.* Regardless of whether objective relative frequencies can be shown to have existed in the past and/or whether subjective probabilities exist today, the economic agent believes that during the time between the moment of choice and the payoff, unforeseeable changes will occur. At this time, the decision-maker believes that no information regarding future prospects exists today and therefore the future is not calculable.

Davidson (1996, p. 482) describes *fundamental uncertainty* about the future course of events in terms of “the absence of governing ergodic processes,” or as a creative, transmutable environment, calling this situation nonergodic. In nonergodic environments, even if individual agents have the ability to collect and successfully process all the information relating to past and current outcomes, this existing market information still cannot provide reliable data for forecasting future outcomes and learning about the future.

The above shows that some weighty arguments have been put forward against the ergodic hypothesis. In order to understand “non-routine” decisions in the real world and their consequences, as well as unforeseeable changes in behaviour, we have little use for ergodic probability distributions. In a world full of true uncertainty – where real historical time determines the course – the probabilities that govern the past are not those that will govern the future. In their *denial of the timelessness argument* in neoclassical economics, the post-Keynesians might have relied on the robust views of Hicks (1979, p. 121), who spoke dismissively of the applicability of the probability calculus to economics, as the quotation below illustrates:

“When we cannot accept that the observations, along the time-series available to us, are independent [...] we have then, in strict logic, no more than one observation, all of the separate items having been taken together. For the analysis of that the probability calculus is useless; it does not apply. [...] I am bold enough to conclude, from these considerations, that the usefulness of ‘statistical’ or ‘stochastic’ methods in economics is a good deal less than is now conventionally supposed. [...] we should always ask ourselves, before we apply them, whether they are appropriate to the problem at hand. Very often they are not. [...] The probability calculus is no excuse for forgetfulness.”

If the economic process moves in historical time and the process is therefore not stationary, and if its distribution function is not independent of calendar time, and if the direction of change in the distribution function is likewise not independent of calendar time, then the environment is clearly nonergodic; furthermore, any estimated statistical average can be persistently and non-systematically *different* from the future time averages actually occurring in the economy. In the above-quoted work, Hicks (1979) rejected the belief in the dominance of ergodic processes in economic phenomena, declaring that “economics is in time, in a way that natural sciences are not. All economic data are dated; so that inductive evidence can never do more than establish a relation which appears to hold with the period to which the data refer.” (op. cit. 1979, p. 38) To this Davidson (1978) adds that, unlike the physical sciences, where the date at which an experiment is made or repeated is irrelevant for the significance of that experi-

ment, economics is a process in historical time, where the random functions are not in a state of statistical (or experimental) control.

In keeping with the opinion of Hicks (1979, p. 3), we can say that timeless, stable relationships are in reality only conceivable in laboratory experiments in the natural sciences. Or we could also propose that through actions, human existence alters the conditions established in the past. Arguments such as this highlight the limits of application of the ergodic hypothesis to economics. In so far as we assume the economic environment to be nonergodic, Davidson's explanation of the forecastability of economic time series (1988, p. 85) is worthy of attention:

“Time series data of spot financial prices are merely the stringing together of momentary, hourly, daily, etc. historical price observations [...] which primarily reflect the actions of those who are attempting to outguess average opinion. No wonder that, despite the billions of man-hours [...] spent searching [...] for systematic repetitive patterns [...], no one has ever succeeded. These spot price movements reflect the nonergodic ebb and flow of speculative expectations.”

In this way, we may reach the conclusion that, due to the liquidity conferred on equities by financial markets, the resultant “long-term flow of expected yields cannot be the sole consideration in determining value or in the shaping of agents' willingness to own” (Davidson, 1978, p. 204). The same author (1988, pp. 84–85) takes the argument further, arguing that, in the case of assets traded on organized financial markets for minimal transaction costs, the price that the agent believes is tomorrow's price is the sole relevant consideration that determines the price it is worth paying for the asset today. The price that may belong to the future, as well as the asset's long-term yield outlook, enters the picture only indirectly and dependently, i.e. to the extent to which it is believed it will influence the price which the agent will pay tomorrow. On this basis, it is an irrational pursuit on the part of the equities market player to be concerned with anything but the short term.

With respect to the latest financial crisis, *Peters* (2009) stressed that the real breach in thinking on uncertainty was caused by the rational expectations hypothesis and the role of ensemble averages.

“In an investment context, the difference between ensemble averages and time averages is often small. It becomes important, however, when risks increase, when correlation hinders diversification, when leverage pumps up fluctuations, when money is made cheap, when capital requirements are relaxed. If reward structures – such as bonuses that reward gains but don't punish losses, and also certain commission schemes – provide incentives for excessive risk, problems arise. This is especially true if the only

limits to risk-taking derive from utility functions that express risk preference, instead of the objective argument of time irreversibility. In other words, using the ensemble average without sufficiently restrictive utility functions will lead to excessive risk-taking and eventual collapse." (op. cit. p. 41)

Peters' discussion is a good example of how thinking about uncertainty in the interpretation of "rational expectations" and "ensemble averages" has seriously adverse repercussions for the financial system.

With their criticism of the ergodic hypothesis, the post-Keynesians, eminent among them Davidson, have come to the conclusion that "important decisions involving production, investment and consumption activities are often taken in an uncertain (nonergodic) environment" (Davidson, 2007, p. 87). By "uncertain," most economists mean that "it is not possible to give a well-defined probability distribution" (Sauter, 2014, p. 39).¹⁰

Theoreticians of mainstream neoclassical economics reject the post-Keynesian concept of ergodicity, and Davidson's ergodic/nonergodic dichotomy as a part of this. It is worth noting that a number of eminent economic thinkers have expressed agreement with Davidson's concept of nonergodic uncertainty. What is particularly worth emphasizing is that *their sympathy extends explicitly to the nonergodic mode of approach*, and not to the declaration of the ergodic axiom as the foundation of neoclassical theory.¹¹

By criticizing the ergodic hypothesis, the post-Keynesians, and particularly Davidson, confront the predictive paradigm of neoclassical economics and the rational expectations hypothesis. The former states that the economic system as a whole is to be explained using models that provide testable predictions (Friedman, 1953). The rational expectations hypothesis states that market prices are reliable indicators of fundamental value and available information, and are

¹⁰ An alternative definition of uncertainty from the natural sciences would define it as the difficulty of making a precise measurement. Here, uncertainty depends on the precision of the instrument used to make the measurement.

¹¹ Regarding this, DAVIDSON (2007) stated: "Three Nobel Prize winners in economics have recognized the importance of my post-Keynesian analysis associating Keynes' concept of uncertainty with nonergodic stochastic processes. After reading my 1982–83 article on the fallacy of rational expectations, Nobel laureate Sir John Hicks wrote me a letter dated February 12, 1983, in which he stated: 'I have just been reading your RE (rational expectations) paper ... I do like it very much ... You have now rationalized my suspicions and shown me I missed a chance of labelling my own view as nonergodic. One needs a name like that to ram a point home.' In a letter dated May 21, 1985, Nobel laureate Robert M. Solow wrote: 'Let me first say that I always admired that article of yours on nonergodic processes and I thought it was right on the button.' Furthermore, Nobel laureate Douglass North (2005, p. 19) explicitly cites my emphasis on nonergodic analysis with changes in future economic events."

at the mercy only of random fluctuations. Davidson (2010, p. 17) sees in both hypotheses the oblique assumption that the economic world is “ergodic” (following random events), that the future is merely a statistical shadow of the past, and that in the final analysis expectations of the future may commonly take the form of calculated probabilities.

Neoclassical economics is firmly committed to model-based forecasting. *Hausmann* (1994, p. 181) stresses that in one of the discipline’s canonical texts, *Friedman* (1953) argues that the task of positive economics is to provide a system of generalizations that can be used to make correct predictions, and that its status as a science depends on its ability to test these predictions for their precision, scope and conformity with experience.

In recent times (particularly on the back of the financial crisis) scepticism has increased with respect to the all-pervading credibility of the ergodic concept. According to *Bronk* (2009, p. 27), economics cannot disregard uncertainty, unless it wishes to restrict itself to the ergodic system in the narrow sense, where the future is a shadow of the past. In the field of financial theory, failures of forecasting connected to the crisis have called the authenticity of models into question. As *Hodgson* (2011, p. 191) states: “an underlying error lies in overestimating the importance and possibility of prediction.” In his view, representatives of economics should not regard it as a loss of scientific authenticity if they base their work on models that tend more to explain and simulate, rather than to forecast precise outcomes.¹²

Scepticism reaches its peak when it emerges that the infinite and unpredictable future states of economic processes cannot be construed on the assumption of ergodicity, since it is impossible to calculate the space average if the very elements of that space average are unknown. The potential congruence of the space and time averages is – in this case – impossible to determine, and is connected to the absence of ergodicity. *Kirstein* (2015) speaks of an outright “ergodic fallacy,” comprised of the mistaken belief in a causal relationship, when the relationship is either non-existent or is constantly changing. The environment is embedded in real time, and processes thus occur in historical time. For this reason, *Kirstein* believes that in such an environment the analyst observes a process within a finite time window. The analyst has no knowledge or data about any time periods prior to this observation window, or simply has no interest in those time periods, so that nothing before the observation window is

12 As an example, *HODGSON* (2011, p. 192) mentions biology, as a prominent branch of the modern natural sciences which rarely attempts to forecast the future with any precision, instead recognizing the central importance of random mutations, the threshold effect and increasing returns. In his view, economics may draw inspiration from biological metaphors in embracing complexity and downgrading prediction, in favour of causal explanation as the primary goal.

subject to analysis. The analyst thus falls prey to the ergodic fallacy, believing it possible to infer a (spurious) causal relationship from his observed time period with respect to the future.

IS THERE A CONNECTION BETWEEN KEYNES' CONCEPT OF UNCERTAINTY AND THE ERGODIC AXIOM?

For the post-Keynesians, in a context in which time is historical, economic agents cannot decide their future actions through analysis of statistical data series, or based on sentiments or beliefs justified by past experience. In their view, decision-making takes place in an environment of true uncertainty. Davidson's model (1991) is based on the demarcation of ergodic-stochastic and nonergodic-stochastic processes, emphasizing the incalculable nature of uncertainty.

It is reasonable to assume that Davidson was motivated to shape the nonergodic argument to use in his struggle against the rational expectations hypothesis of Lucas and *Sargent*. Curiously, the latter were not concerned with whether the axiom of rational expectations was true, since the "axiom" was something they presumed, and cannot be tested. *At the focus of debate for decades is the question of what attitude Keynes might have taken to the ergodic axiom and the rational expectations hypothesis.* Besides being anachronistic, the question is extraordinarily difficult to answer due to its complicated background. Davidson (1972; 2007; 2009; 2011) made numerous retrospective attempts to hypothetically demonstrate Keynes' implicit rejection of the relevance of ergodicity. The most intriguing question may be whether Keynes could have known about the debate over the ergodic hypothesis in the 1920s and 1930s. Keynes never made mention of this in any of his writings, even when the role of probability was the topic of debate. Keynes' thinking was very close to the arguments that arose at that time in discussions of ergodicity (it is sufficient merely to cite Keynes' 1921 work on probability). Consequently, even with hindsight, it is not possible to answer the question of whether Keynes was familiar with the ergodic principle; to determine whether he did not wish to respond to it, or had not even encountered this attribute of regularity in dynamical systems. In every direction regarding component issues of ergodicity, Keynes was in opposition to his contemporaries. The focus of debate with Tinbergen was the question of homogeneity of economic data series, as well as how concepts applied in physics could be transferred to economics. Keynes was sceptical that non-homogenous data series could represent any kind of probability distribution. He also had his doubts regarding the relevance of adopting concepts directly from physics. On the other hand,

he may have been influenced by Ehrenfest, who first suggested in his doctoral thesis on physics that his ideas might be applicable in economics too.

This opposition is also recognizable with respect to the classical axiom of “perfect knowledge.” Davidson (2011, p. 35) wrote that early 19th century economists (such as *Ricardo*) assumed that every decision-maker has perfect knowledge of the future. In this instance, it is not necessary to use probabilities to develop actuarial knowledge. By the beginning of the 20th century, classical mainstream theory had abandoned the perfect knowledge axiom. Instead, these theorists identified knowledge about the future with probabilistic risk, in that the latter can be calculated from existing data. In other words, Davidson stresses, this conversion of risk calculations to “actuarially” certain knowledge about the future required the classical economic theory of Keynes’ time to implicitly assume that processes in the economy were generated by what modern probability theory calls an ergodic stochastic process.

Keynes’ work (1921) presents his views on expectations and knowledge to the fullest extent. The most important thing here is that he adopts a new position with respect to proof, as clearly evident in his assertion that “the terms ‘certain’ and ‘probable’ describe the various degrees of rational belief about a proposition which different amounts of knowledge authorise us to entertain” (op. cit. p. 3). At the same time, Keynes was also cautious, leaving little doubt that, while he rejected total scepticism (which he felt to be overly pessimistic), he did not fully share the claims of believers in probability. The most relevant arguments in the present context embrace Keynes’ doubts about whether numerical probabilities can be assimilated or an order of magnitude given to probabilities. In the case of numerical probabilities, most experiments apply a simple formula, incorporating a large number of instances and reinforcing the generalization; Keynes, however, rejected these formulas.

Keynes never explicitly declared that his general theory would have demanded rejection of the ergodic axiom. Instead he was merely against the application of probability analysis in forecasting the future, while also declaring that there is no basis for developing scientific calculations supporting actuarially certain knowledge of future outcomes. Explaining the general theory, Keynes came to the conclusion that, in his opinion, the classical axiom must be rejected, allowing for the impact of uncertainty influencing entrepreneurial decision-making. Keynes (1937) wrote in explicit terms when it came to discussing classical theory:

“...facts and expectations were assumed to be given in a definite and calculable form; and risks [...] were supposed to be capable of an exact actuarial computation. The calculus of probability [...] was supposed to be capable of reducing uncertainty to the same calculable status as that of

certainty itself. [...] I accuse the classical economic theory of being itself one of these pretty, polite techniques which tries to deal with the present by abstracting from the fact that we know very little about the future. [...] I think [the classical economist] has overlooked the precise nature of the difference which his abstraction makes between theory and practice, and the character of the fallacies into which he is likely to be led.” (Keynes, 1937, pp. 212–215)

In this statement, Keynes explicitly indicated that in his “General Theory” he was obliged to reject the assumption of 20th century classical theory that decision-makers use probabilities to calculate actuarially certain knowledge of future events. In his explanation (1937) of his general theory (1936), Keynes specifically attacks the 20th century classical theory that a decision-maker is capable of preparing an “exact actuarial computation” of future events, reducing uncertainty to the “status of certainty.” It is clear from Keynes’ explanation (1937) that he rejected the assumption of classical theorists that the future is actuarially calculable through analysis in the context of (known) probability theory.

Davidson and the post-Keynesians recognize that Keynes could not have known of the ergodic/nonergodic distinction, and yet they have claimed that Keynes implicitly drew this distinction when formulating his ideas in relation to uncertainty, and that “in retrospect, therefore, we can only seek to reinterpret Keynes’ fine intuition of the distinction” (Davidson, 1982–83, p. 188; 1995, p. 19; 2006, p. 190). Since the classical economists of Keynes’ time were using probability analysis to calculate an actuarially certain future, then – according to Davison – they must have assumed that the economy was being generated by an ergodic stochastic process, even if they did not use the terminology of stochastic (probability) theory. In this analysis, there is no scientific basis for using existing facts and data to calculate actuarial knowledge of future economic events if the facts and data generated by the stochastic process are nonergodic. Consequently, *Keynes’ concept of uncertainty, supposing a system where he specifically states there is no scientific basis for producing an actuarially certain computation regarding future economic events*, is logically consistent with the classification implication in stochastic theory that economic facts are generated by a nonergodic stochastic process.

In Keynesian thinking, uncertainty is not related to probable knowledge, but precisely to the lack of it. Uncertainty corresponds to a situation where probabilities cannot be determined numerically, or it is not possible to more or less compare them to other probability relations. Thus Keynes indicates that by “uncertain knowledge,”

“...I do not mean merely to distinguish what is known for certain from what is only probable. The game of roulette is not subject, in this sense, to uncertainty; nor is the prospect of a Victory bond being drawn. Or, again, the expectation of life is only slightly uncertain. Even the weather is only moderately uncertain. The sense in which I am using the term is that in which the prospect of a European war is uncertain, or the price of copper and the rate of interest twenty years hence, or the obsolescence of a new invention, or the position of private wealth-owners in the social system in 1970. About these matters there is no scientific basis on which to form any calculable probability whatever. We simply do not know.” (Keynes, 1937, pp. 213–214)

Elsewhere he writes:

“The outstanding fact is the extreme precariousness of the basis of knowledge on which our estimates of prospective yield have to be made. Our knowledge of the factors which will govern the yield of an investment some years hence is usually very slight and often negligible. If we speak frankly, we have to admit that our basis of knowledge for estimating the yield ten years hence of a railway, a copper mine, a textile factory, the goodwill of a patent medicine, an Atlantic liner, a building in the City of London amounts to little and sometimes to nothing; or even five years hence.” (Keynes, 1936, pp. 149–150)

Keynes’ explanatory paper (1937) argued that our knowledge of the returns on investments is “uncertain,” and uncertainty must be differentiated from what is probable. Interpreting situations of uncertainty according to the above, Keynes’ next question is how one should behave under such circumstances if one wishes to remain a rational *homo economicus*. To this Keynes replies:

“Knowing that our own individual judgment is worthless, we endeavour to fall back on the judgment of the rest of the world which is perhaps better informed. That is, we endeavour to conform with the behaviour of the majority or the average. The psychology of a society of individuals each of whom is endeavouring to copy the others leads to what we may strictly term a conventional judgment.” (Keynes, 1937, p. 214)

The most important message of the latter quote from Keynes is that *information is incomplete and uncertainty about the future generally makes it impossible for entrepreneurs to form rational expectations, a fact that carries definitive significance with regard to their investment decisions*. The upshot of all this is that entrepreneurs’ expectations are largely conventional, and as such, remain at the mercy of alternating waves of optimism and pessimism. This is what is meant by Keynes’ famous “animal spirits,” explained in his own words as follows:

“Even apart from the instability due to speculation, there is the instability due to the characteristic of human nature that a large proportion of our positive activities depend on spontaneous optimism rather than on a mathematical expectation, whether moral or hedonistic or economic. Most, probably, of our decisions to do something positive, the full consequences of which will be drawn out over many days to come, can only be taken as a result of animal spirits – of a spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities. [...] Thus if the animal spirits are dimmed and the spontaneous optimism falters, leaving us to depend on nothing but a mathematical expectation, enterprise will fade and die; – though fears of loss may have a basis no more reasonable than hopes of profit had before.” (Keynes, 1936, pp. 161–162)

The same thought appears in what Keynes sees as the role of professional analysts dealing with stock market investments:

“...most of these persons are, in fact, largely concerned, not with making superior long-term forecasts of the probable yield of an investment over its whole life, but with foreseeing changes in the conventional basis of valuation a short time ahead of the general public. They are concerned, not with what an investment is really worth to a man who buys it ‘for keeps’, but with what the market will value it at, under the influence of mass psychology, three months or a year hence.” (Keynes, 1936, p. 154)

Looking at the basis of Keynes’ interpretation of probability from the logical point of view, probability is not subjective. In other words, probability does not depend on human caprice. A proposition does not become probable because we think it to be so. Once the facts are given that determine our knowledge, what is probable or improbable has been fixed objectively and is independent of our opinion. According to Keynes (1921, p. 4): *“The Theory of Probability is logical, therefore, because it is concerned with the degree of belief which it is rational to entertain in given conditions, and not merely with the actual beliefs of particular individuals, which may or may not be rational.”*

As Gillies (2003) notes, in the above-quoted passage Keynes regards probabilities as objectively fixed, but he does not use the attribute “objective” in reference to things in the material world. When Keynes uses the term, he refers to the objective in a world of abstract ideas. The next question we might ask regarding Keynes’ approach, according to Gillies, is how we may obtain knowledge about this logical relation of probability. Keynes’ answer is that we get to know at least some probability relations by direct observation or immediate logical intuition. A problem that arises here is how we can assign numerical values to these probabilities. Keynes believes that this is possible only in some cases, and on this

point states (Keynes, 1921, p. 44): “In order that numerical measurement may be possible, we must be given a number of equally probable alternatives.” This a priori principle is called by Keynes the “principle of indifference.”

When applying the probability calculus and a priori conventions, the problem is that convention is exposed to “waves of optimistic and pessimistic sentiment” (Keynes, 1936, p. 154); as “the mass psychology of a large number of ignorant individuals is liable to change violently as the result of a sudden fluctuation of opinion due to factors which do not really make much difference to the prospective yield; since there will be no strong roots of conviction to hold it steady” (op. cit. p. 154). Factors hitherto disregarded by convention may thus reappear – exercising a dramatic impact.

Keynes’ theory does not demand that we assume the existence of individuals who have no knowledge of how scientific and statistically significant calculations are made with respect to the forecasting of future events, in areas where facts are generated by ergodic stochastic systems, such as – for example – the insurance sector and many serious sciences. It was Keynes himself, in his work on probability (1921), who raised the matter of insurability, recognizing that there are cases when such probabilities are known. Accordingly, *Keynes recognized that it is not epistemological uncertainty but ontological uncertainty that arises here, separating insurable (forecastable) future economic events from uncertain ones.*

In the language of probability theory, Keynes claims that existing economic data cannot be used to calculate the actuarial price of copper 20 years hence, which means that if economic theory is sufficiently suited to the real world of experience, then – according to Davidson (1982–83) – Keynes must reject the ergodic axiom. *Davidson retroactively projects his own ergodic/nonergodic dichotomy onto interpretation of Keynes’ viewpoint, although he knows that Keynes never thought in terms of this division.* Application of the ergodic axiom leads to the assumption that people can have actuarial knowledge of the future; when Keynes states that we cannot know the future, and when classical theory leads to all manner of “falsities,” then it is merely a logical alternative for Keynesian economic theory to concede that existing economic data (facts) are generated by a nonergodic stochastic process. And if data are generated by a nonergodic system, then there is no scientific method permitting the future to be actuarially calculable from the existing database. The future must be uncertain, irrespective of just how much historical data has been gathered and analyzed.

Keynes disputed the views of all those who believed in the existence of a general economic regularity. His philosophy carries a negative message against acceptance of the concept of the “ergodic nature” of the economic universe. The focus

for him was on the question of homogeneity of data series.¹³ In his debate with Tinbergen, Keynes (1939) questioned the constant nature of estimated coefficients. He claimed that application of regression analysis to generate parameters, and then treating these as constant, is fundamentally flawed: "...there is no reason at all why they should not be different every year," since we know that many economic relations are not homogenous in time (op. cit. p. 560). He also criticized the ad hoc character of some variants of quantitative modelling: "With a free hand to choose coefficients and time lag, one can [...] always cook a formula to fit moderately well a limited range of past facts. But what does this prove?" – asked Keynes. To this he added as a third consideration that "there are important influences that cannot be reduced to statistical form," because if we attempt this then we obtain a "false precision" as a result (op. cit. p. 561).

The post-Keynesians take the view that economic processes are not stationary and move in historical time, since actions in society have a direct influence on these processes. In this they follow the opinion of Keynes, who held that economic time series are not stationary given that the economic environment is not homogenous over a given period of time. Solow (1985, p. 328) asserted something similar when he wrote that "...much of what we observe cannot be treated as the realization of a stationary stochastic process without straining credulity."

Keynes did not believe a central mind would be able to entirely overcome uncertain situations. He took the view that the economy is too complex to lend itself completely to modelling, from which he came to the conclusion that economic theory is the simplified presentation of close relations, and does not present the entirety. In his formalized philosophy of probability, Keynes presented total uncertainty, which is in reality one extreme variant of the possible states of knowledge. According to Keynes, decisions under conditions of total uncertainty are taken according to whim or chance, although the main theme of his standard work discussing probability was examination of the potential for rational decision-making in possession of non-quantitative information. Keynes' starting point was that the basis for knowledge on which estimates of future returns must rest is extremely precarious. He held this to be an implicit axiom of classical theory applying to the self-regulating economy, where economic agents have access to statistical probability pertaining to their investment outcomes. Regarding risk, he accepted the assumption that it can be the basis for precise, actuarial calculation. At the same time, he stated that "we simply do not

13 KEYNES' favoured method for testing homogeneity – adopting the method of LEXIS (1879) – was to break down time series into subgroups; estimating coefficients for each, he examined whether they displayed stability over time. Although this kind of stability is not entirely the same thing as stationarity, Keynes saw positive correlations, asserting that the greater the degree of homogeneity connected to the time series, the greater the stability and the stronger the stationarity.

know” what return our investment will bring in five or ten years or even further forward. Investments that promise a return at a comparably distant or indeterminate distant point in time are to be regarded as the work of fate.

Projecting Davidson’s own notion of nonergodicity (2006, p. 150) onto the Keynesian concept of uncertainty, Keynes emerges as a *prototype* of the nonergodic concept:

“Keynes rejected this view that past information from economic time-series realizations provides reliable, useful data which permit stochastic predictions of the economic future. In a world where observations are drawn from a nonergodic stochastic environment, past data cannot provide any reliable information about future probability distributions. Agents in a nonergodic environment ‘know’ they cannot reliably know future outcomes.”

The foregoing may convince us that Keynes – *expressis verbis* – did not reject the ergodic hypothesis, and did not base his concept of uncertainty on the idea of nonergodicity, and this is still true even if the Keynesian sphere of thought bore a close resemblance to this theory.

An unfairly forgotten thinker:

Shackle, the first adherent of the Keynesian concept of uncertainty

Following Keynes’ approach to uncertainty, Shackle (1949; 1955) developed the principle of cruciality, in order to differentiate the uncertain world containing historical time from ergodic processes. When agents make crucial decisions, they necessarily destroy any kind of ergodic stochastic process that may have existed at the point the decision was made. According to Shackle (1955, p. 6), an agent commits himself to crucial decision-making in situations when “the person concerned cannot exclude from his mind the possibility that the very act of performing the experiment may destroy forever the circumstances in which it was performed.” Shackle had already established earlier (1949, p. 6) that economic decisions of a non-routine nature carry crucial significance as unique “experiments.”

Shackle asserted that individuals are incapable of quantifying all possible eventualities or states of the world. In this sense, an individual agent is only able to make decisions if they create their own set of choices, but this implies that the choice set thus created is necessarily incomplete. This is the main analytical point of Shackle’s theory. On this basis Shackle constructed a formalized model, the goal of which was to simultaneously grasp the mental processes and the non-recurring, irreversible nature of economic decisions in the present.

Shackle analyzed the decision-maker, typically an entrepreneur, who must choose from among alternative “sequels” in the course of action, based on two elements: the possible gains and losses embedded in a sequel to a specific action, called “face values,” and a valuation of the “possibility” of gains and losses, called the *potential surprise*. The latter element represents the degree of disbelief, or implausibility of the hypothesis that supports the sequel, ranging from 0 (absence of disbelief or zero potential surprise) to a maximum value expressing impossibility (absolute disbelief or maximum potential surprise).

Explaining Shackle’s essential thesis (1955), Davidson and Davidson (1984, pp. 329–330) note that agents themselves must discover or shape the future through their actions carried out within the framework of existing and developing organizations. Elsewhere Davidson (1996, p. 482) stresses that individuals recognize that “the external reality in which they operate is in some, but not necessarily all, economic dimensions not only uncertain but also transmutable or creative.” This creative economic reality brings with it an uncertain future which can be permanently shaped by individuals and groups, often in ways not completely foreseeable by the creators of change. According to Shackle (1955), decision-making in situations where information gathered from the past represents an inadequate basis for evaluating future outcomes will inevitably be of a creative nature. *In all situations where decisions are irreversible, economic prospects change forever and the circumstances of the decision are non-repetitive, making the choice crucial.*

Based on Shackle’s explanation, Davidson (1980, p. 102) writes that “crucial choice involves, by definition, situations where the very performance of choice destroys the existing distribution functions. Since the resulting outcome of the choice must occur at a point of time following the performance [...], the future is created by crucial choice decisions [...] If important decisions regarding the accumulation of wealth, the possession of liquidity, the commitment to a production process with significant set-up costs and gestation period, etc. are crucial, then ‘the future waits, not for its contents to be discovered, but for that content to be originated’.”

Shackle (1955) emphasized that decision-making in situations where information is gathered from the past relies on an inadequate basis for evaluating future outcomes since the basis will inevitably be of a creative nature.¹⁴

When a decision is irreversible, when the economic environment is forever changing, and when the circumstances of the decision are non-repetitive, then

¹⁴ SHACKLE (1974) makes the following observation on the meagre relevance of past events: “By the kaleidic theory I mean the view that the expectations which [...] are at all times so insubstantially founded upon data and so mutably suggested by the stream of ‘news’ [...] that they can undergo complete transformation in an hour or even a moment...” (op. cit. p. 42)

the decision is crucial. The novelty and creativity that evolves in connection with such decision-making is linked in a complex way to uncertainty. Economic agents must make decisions since often there is no precedent. This is the underlying principle of Shackle's theory, which supports and links the concepts of cruciality and uncertainty. Shackle's arguments clearly follow the line of thinking pursued by *Knight* (1921). In Shackle's view, use of the probability calculus in analysis of decisions under conditions of uncertainty was inappropriate because the conditions demanded for its application simply did not exist in many relevant economic contexts. His contention was that *in reality individuals do not have complete knowledge of the structure of the world – knowledge which is assumed by probability calculus. Individual choices are made between alternatives that are subjective representations of alternative future sequels to action, and not choices between future sequels themselves.* In Shackle's own words, "choice is among imagined experiences," a view which implies that individuals are not given an exhaustive list of the alternatives from which their choice is made.

Dispensing with probability, Shackle turned to *potential surprise*. He argued that instead of clarifying the potentially uniform influence of many important decisions, better appreciation needed to be given to the individual's cognitive status. For Shackle, a relevant theory of decision-making demanded explicit recognition of the individual's mental activities, such as *imagination* and the accommodation of potentially unknown events. To achieve this, he detached his own theory from the application of probability. Thus Shackle (1955; 1972) became the main developer of the implications of non-quantifiable uncertainty. He took an uncompromising stance in accepting the ontological nature of Keynesian uncertainty, strongly emphasizing its role in investment decisions, and the unpredictability and peculiar variability of investment outcomes. He saw this uncertainty as tied to the creativity and free will of the investor and to their ability to create a new reality from nothing, which is constantly changing and upsetting previous realities as "potential surprise" emerges. This prompted him to reject the concept of cognitive equilibrium and to declare that reality is "kaleidic" (Shackle, 1974) – constantly changing and swirling like the colours of a kaleidoscope.

It is a peculiar circumstance that, according to *Fitzgibbons* (1996), Shackle sees *nihilism* in Keynes' approach, yet – in his view – the goal of Keynes' treatise on probability was not to prove that "we all float on the Great Sea of Unknowing" (Fitzgibbons, op. cit. p. 75). Keynes rejected the doctrine of radical uncertainty, conceiving a new kind of logic that differed from formal logic, and which was applied when knowledge was partial and non-quantitative in nature. Keynes defined probability to include a broad range of information, extending from

luminous certainty to complete ignorance, arguing that everyone uses the logic of probability all the time. The peculiarity and oddness of this situation is reflected in the fact that *Coddington (1982) respected Shackle as a proven exponent of the logical implications of Keynesian uncertainty, even as he rejected Shackle's nihilism.* The main issue in the nihilism argument formulated by Coddington lies in whether there can be any predictability if there is such profound uncertainty. An answer to this lies in the cruciality argument put forward by Shackle (1955), who takes the view that potential surprise and uncertainty may arise only with crucial decisions such as major capital investments, or the choice of a career for an individual. Routine decisions, on the other hand, may be governed by ergodicity and can be somewhat predictable, including much of consumption behaviour. *Given that the essential creativity of the capital investment process driven by the "raw force of nature" is inherently surprising and uncertain, it provides an ontological foundation for uncertainty.*

Among interpreters of Keynes, Shackle is known as the one who placed the greatest emphasis on the unpredictability of the future as the basis for individual decision-making. Coddington (1982, p. 480) formulated a very strict opinion with respect to the post-Keynesian standpoint on uncertainty in economic decision-making in general, and Shackle's opinion in particular, by saying that "there are those who have singled out this uncertainty theme not only for its especial importance, but also for its potential for analytical subversion." He described Shackle's all-embracing subjectivism as "consistent but analytically nihilistic" (*ibid.*).

Shackle was the first to demonstrate *the vital role of imagination* under conditions of economic uncertainty, a role concisely expressed in the aphorism: "Valuation is expectation and expectation is imagination" (Shackle, 1972, p. 8). Shackle stated that, given we have only fragmented and confusing evidence about what tomorrow brings, we must build a picture – with the help of imagination – of what may come. In an economic context, he wrote: "Economic choice does not consist in comparing the items in a list, known to be complete, of given fully specified rival and certainly attainable results. It consists in first creating, by conjecture and reasoned imagination on the basis of mere suggestions offered by visible or recorded circumstance, the things on which hope can be fixed." (*op. cit.* p. 96)

The theory behind the choice of uncertainty was mostly motivated by the need to view uncertainty as uncertainty, and not risk; the consequence of this was that it became necessary to introduce something to replace probability descriptions of the likely, uncertain outcomes arising from competing, alternative strategies or choices. Shackle advanced several reasons to demonstrate that the probability calculus is not a suitable indicator of uncertainty. The most impor-

tant boost to his objection was that *economic choices are generally individual, isolated decisions and choices; they are not parts of a long series of similar “experiments,” so that the application of probabilities based on rates of frequency is problematic.* Decisions are generally individual events, since they differ from one another from case to case.

In his work on *the role of time in economics*, Shackle (1958) not only expressed doubts with respect to the validity of formal, mechanical time dynamics models, but his observations, if correct, excluded any type of forward-looking economic model, except for that directed at the next period. Predictions could be made for one period ahead, given the state of current expectations and intentions. As the next period runs out, market participants *form new expectations*, not necessarily connected to something that has already happened earlier, and it is possible that their actions are based on something new in the market (for example, potentially involving a real investment in innovation). Shackle’s opinion is succinctly expressed in the following quote:

“In the classical dynamics of the physicist, time is merely and purely a mathematical variable. The essence of his scheme of thought is the fully abstract idea of function, the idea of some working model or coded procedure which, applied to any particular or specified value or set of values of one or more independent variables, generates a value of a dependent variable. For the independent variable in a mental construction of this kind, time is a misnomer [...] The solution to the differential equation, if it can be found, is complete in an instantaneous and timeless sense.” (*Time in Economics*, pp. 23–24)

The implication of Shackle’s work, that economics cannot be a predictive science, has not occasioned unanimous agreement. From the early 1960s onwards, *Shackle’s intellectual influence waned*, in no small part due to the appearance of the rational expectations hypothesis and its later ascent to a dominant position in theory. Despite this, based mainly on post-Keynesian and heterodox tradition, he has come to be regarded as a representative of decision theory capable of grasping the true and authentic nature of uncertainty.

Uncovering the ontological foundations of uncertainty

In connection with the subject of our investigation, O’Donnell (2013) alludes to three fundamental distinctions: ergodic/nonergodic, risk/uncertainty, and immutability/mutability. Ergodic realities belong to the domain of risk and immutability, while nonergodic realities fall within the sphere of irreducible uncertainty and mutability. In this approach, ontology is a necessary and satisfactory

condition of the existence of irreducible uncertainty. O'Donnell believes that the nature of the universe determines whether risk or irreducible uncertainty prevails, and not the cognitive characteristics of individuals. *Ontological characteristics are primary and permanent, while cognitive attributes are secondary and eliminable.* If the external economic reality is ergodic (and thus immutable), then no (business or social) practice can be pursued that would alter predetermined future outcomes.

Davidson and Davidson (1984, pp. 329–330) determined that economic agents must discover or shape the future for themselves through their actions carried out within the framework of existing and developing organizations. Individuals recognize that the environment in which they make decisions does not contain governing ergodic processes in some dimensions, meaning that it is uncertain, and thus transmutable or creative. This economic reality brings with it an uncertain future which can be permanently shaped by individuals and groups.

Assuming fundamental uncertainty, future states of the world cannot be specified, since they are taking shape now and in the future. This suggests that future states cannot be anticipated. What has happened in the past, or is taking place in the present, will not necessarily occur in the future as well. Individual agents are ignorant of available courses of action or future states of the world as a consequence of the irreversible and open-ended nature of time and because the future is transmutable, and not because of the limited capabilities of economic agents. Dunn (2000) emphasizes that individuals are builders of the future. In an uncertain world, the future – prior to its unfolding – cannot be known, irrespective of the calculating capabilities attributed to individuals. It cannot be known *ex ante* how any story will pan out, no matter the amount of information and calculating capacity of the decision-maker; the future can never be predicted *ex ante* with (probable) certainty. Individuals and groups of individuals shape their own stories, but the story that takes shape is not necessarily pre-planned. Davidson and Davidson (1984, pp. 329–330) emphasize that, in an uncertain world, rational agents recognize that the future may differ significantly from past experience and present expectations. In a nonergodic world, where the past does not provide a guide to the course of future events, agents are truly uncertain if information does not currently exist that will enable them to discover the future. Decisions have to be made and choices must be genuine. As a result, agents will have to invent or create the future by their own actions within evolving and existing organizations.

Despite the strength of scepticism along the Keynes–Hicks–Shackle intellectual axis with respect to the dominion of ergodic processes in economics (the relevant *terminus technicus* being spoken or unspoken), representatives of mainstream economics were inclined to remain attached to their belief in

the ergodicity of economic phenomena. On the other hand, many believed that economics could not be simply translated into a science identifying with the immutable laws of the natural sciences. As we have seen in the foregoing, uncertainty is an overarching concept at the centre of Keynes' world view. Accordingly, Keynes had doubts with respect to the ability of the individual to precisely forecast the future, particularly in the longer term.¹⁵ On the other hand, however, the modern economic system cannot function without forecasts of the future. We must unconditionally agree with the assertion by Knight (1921) that "at the bottom of the uncertainty problem in economics is the forward-looking character of the economic process itself" (op. cit. p. 237). This is the dilemma from which there is no escape.¹⁶ It is an indisputable lesson that we must make a distinction between cognitive and ontological uncertainty. Mainstream economics underestimates the problematic nature of the connection between theory and reality for both economic actors and researchers, and its representatives disregard the impossibility of knowing the future; a future that is created by innovation itself, and which is strengthened by freedom of choice.

One distinctive feature of mainstream economics is the extent to which economists disregard the centrally important distinction made by Knight (1921, p. 233) between measurable risk (where possible outcomes can be grouped and probabilities calculated) and immeasurable uncertainty (where probability cannot be calculated, since the instance is unique). For Knight, uncertainty was not *some regrettable market disturbance*, but an indisputable part of entrepreneurial activity (op. cit. p. 232). Naturally without uncertainty there could be no profit in a competitive system, since forecastable profit would be rapidly eaten up through competition. Knight demonstrates this epistemological situation as follows:

15 The presence of irreducible uncertainty made KEYNES famously sceptical about the value of econometrics – the use of statistical techniques ("regression analysis") to establish relationships ("coefficients") between independent and dependent variables, which enables someone to predict the value of the dependent variable. The objection of DAVIDSON (1988, pp. 78–79) that we do not live in an economy governed by ergodic processes provides a formal foundation on which basis we can understand Keynes' expression of his conviction that economic uncertainty is fundamental and unmanageable in nature.

16 In neoclassical economics, the discrete generalized linear model (GLM) previously formed the basis of empirical estimation, leading to the application of the method of least squares and the technique of maximum likelihood estimation. This was followed by the technique of estimating generalized least squares (GLS), according to DHRYMES (1974) and THEIL (1971). Generalization of the discrete time estimation approach led to the ARCH and GARCH models (ENGLE and GRANGER, 1987). This approach permitted the modelling of the degree of nonlinearity, and the attainment of an essentially better accommodation with respect to observed economic time series. Financial economics provides numerous empirical examples of GARCH's capacity to predict volatility, and to compare inherent volatility with the historical volatility of financial variables.

“Profit arises out of the inherent, absolute unpredictability of things, out of the sheer brute fact that the results of human activity cannot be anticipated and then only in so far as even a probability calculation in regard to them is impossible and meaningless.” (Knight, 1921, p. 311)

Epistemological uncertainty is a function of other ontological factors: social reality is multi-faceted, made up of the mutual interconnections between physical constraints, institutional frameworks, individual thought and action, social context, and phenomena emerging on the macroeconomic level. Bronk (2011) emphasizes that there can be issues of ontology – such as the multi-faceted nature of reality and the prevalence of nonlinear functions – that impact on the degree of epistemological uncertainty, in the sense of weakening the basis for precise prediction. *Ontological uncertainty refers to indeterminacy at the level of reality itself, and to the logical unknowability of the basic categories and entities of future reality ahead of its creation.*

According to Bronk (2011, p. 9), ontological uncertainty “implies the impossibility of knowing even the categories and possible nature of what has yet to be created or yet to evolve.” As *Buchanan and Vanberg* (1990, p. 323) note, once we see the market as a “creative process,” it makes no sense to see the future as in some sense “out there” waiting to be discovered, nor to use the language of error for forecasts that get overturned as if “correct” forecasts were possible. Naturally, according to Bronk (2011, p. 9), the widespread and revolutionary impact of innovation and imagined futures is “corrosive of the standard notion that forward-looking market valuations can be stable and efficiently priced – that there is a static reality ‘out there’ on which rational expectations will converge in response to competitive pressures.”

To be able to fully understand the barriers to knowledge in economics, and the suitable methods for breaking down some of these barriers, the implications of Romantic post-Kantian philosophy may provide some assistance. The Romantics understood that we never have access to facts without some kind of intermediary, and our sentiments are not simply reflections of an “external” reality. Instead, the world we see is partly the creation of the conceptual structures our consciousness provides. In the terminology of *Wordsworth* (1798), our eyes and ears “half-create” the world we see and hear. Consciousness thus plays an essential and creative role in every empirical observation by providing the framework conditions for interpretation.

Another important implication of Romantic philosophy for economists, as Bronk interprets it (2009, p. 260), is that our theoretical and conceptual frameworks structure our actions as well as our beliefs and sentiments, which thus have the power to transform social reality. Bronk adds: “In other words, the behaviour that economists study is already partly constructed by the socially

formed narratives and economic theories that individual actors have internalized. This means that economists cannot fully explain or predict economic and market behaviour unless they have learned to empathise with (the better to interpret) the various mindsets and conceptual structures that influence actors' beliefs and reasons for action."

In conditions of uncertainty, the expectations on which decisions are based depend upon imagination as well as reason; they are mediated through narratives and histories, and contain sentiments and emotions. Bronk (2009, p. 221) asserts that imagination and creativity are not only a major cause of ontological uncertainty, but are also important tools for describing that uncertainty. As he puts it: "Imagination fills the void left by the indeterminacy created by innovation and the freedom to choose between novel options; it sketches out visions of how the world might be and how we would like it to be. It provides warnings and goals; it constructs possible options. And none of this is antithetical to reason. Indeed, imagination must rely on reason to stress-test its visions of the possible for their likely feasibility and desirability in the light of past experience."

There is no correct vision of the future, since it is still to be determined by innovations not yet conceived and choices and decisions not yet made; in this space of possibilities, current prices can only reflect our best insights, preferred narratives, and fleeting sense of optimism or pessimism. As long as market prices can be regarded as the summation of heterogeneous narratives and perspectives, and of varying emotions and different dreams, then they may at least help us explore emerging patterns in the underlying economy, since multiple reflected perspectives and intuitions are better than one view only.

The Keynesian theory of a transmutable economic reality refutes the classical theory of an immutable economic reality. Keynes' theory can be characterized by total uncertainty, emphasizing *fundamental uncertainty* as its basis in the definition of transmutable economic reality. *Fundamental uncertainty asserts the impossibility of predicting future knowledge, even when actors are utilizing the entirety of present knowledge.* The theory presumes that the economy operates in an uncertain world, and no one uses available data existing today to obtain authentic guidance with respect to the future. In this uncertain environment, it is impossible for actors to predict or see into the future, and they are thus less likely to make correct financial decisions.

Based on the ergodic axiom, and assuming market efficiency, future outcomes are determined by fundamentals the outcome of which cannot be altered by human action. In contrast, fundamental uncertainty states the impossibility of forecasting future knowledge, even when agents are able to make use of the totality of present knowledge. According to *Dequech* (2002, p. 2), fundamental uncertainty implies that some information does not exist at the time of a deci-

sion because the future is still evolving, and cannot be fully anticipated even with a genuine estimate of probability. In other words, much relevant information cannot be known at the time when many important decisions are made. Fundamental uncertainty – in the views of *Dunn* (2001, p. 3) and *Lawson* (1985, p. 913) – occurs in a situation where multiple (more than one) outcomes can be designated with respect to a given decision, although the value and probability of these outcomes cannot be objectively determined in advance. This means that people simply do not know what will happen. Financial decisions rest on beliefs about likely future conditions, but these are beliefs and sentiments that must be based on conditions in the past and present. *Coddington* (1982, p. 480) states that price behaviour may take a false course accordingly, either because present conditions change in a false manner, leading to falsely fluctuating sentiments with respect to future conditions, or because beliefs/sentiments change falsely without the appropriate changes occurring in the basis for the conditions. In an ergodic system according to *Davidson*, there is always some uncertainty about the future.

Davidson (1991, p. 134) defines uncertainty as an environment which emerges in all instances when the decision-maker either cannot think through the entire list of all future consequences, or cannot assign probabilities to all consequences because the evidence is insufficient to establish probability. Proceeding in *Knight's* footsteps towards a clear understanding of true uncertainty, *Dunn* (2001, p. 2) and *Porterfield* (1965, p. 107), for example, distinguish true uncertainty from calculated risk. The concept of uncertainty means more than probabilistic calculated risk. Calculated risk refers to circumstances where multiple outcomes are possible as the result of a given decision, and the value and probability of these can be established in advance; true uncertainty, on the other hand, is not merely a situation in which the probability relation is known, but a set of circumstances where more than one possible outcome is associated with a decision, and the value and probability of these outcomes cannot be objectively determined in advance. *Dunn* (2001, p. 12) mentions that where the past provides only limited and narrow guidance for the future that still needs to be created, crucial decisions can be made, but in this regard agents are uncertain whether there are regularities waiting to be discovered. Even if agents are capable of successfully gathering and processing all the information linked to the past and present, this existing market information does not (and cannot) provide reliable data for predicting future outcomes and learning about the future. *Dunn* (2001, p. 13) emphasizes that if conditions of true uncertainty prevail in certain decision-making areas, then at least some economic processes are such that expectations based on past probability distribution functions can differ persistently from the time averages that will be generated as the future unfolds and becomes historical fact.

The decision-making process is undoubtedly influenced by fundamental uncertainty. Although many have accepted the efficient market theory and the ergodic axiom on which it rests, Keynes questioned the method allowing for the existence of knowledge or fundamentals appropriate for decision-making (Davidson, 1998, p. 3). Based on Keynes' view, certainty in keeping with the concepts of decision-making is impossible – even if there are fundamentals or information permitting an attempt to estimate the objective probability distribution of possible future outcomes, since these could not serve as a sufficient basis for calculable mathematical expectations, and neither could they be connected to the future.

As a believer in the ontological nature of uncertainty, Keynes developed a theory on transmutable economic reality that also provides a distinctive explanation in the context of decision-making on financial markets. Keynes' assumption was that investment decisions and expectations of the future occur in a perpetually transforming economy, i.e. in an uncertain world, limiting the ability of economic agents to predict the future and thereby making it impossible to predetermine the outcomes of financial decisions. When the environment is fundamentally uncertain, then knowledge of the past does not provide sufficient information for prediction of the future, since existing knowledge or fundamentals will not be linked to the future. Decision-making is dominated by “animal spirits”, and preferences in decisions characterized by significant distortion. Consequently, financial markets are needed for the management of fundamental uncertainty, so that market participants can defend themselves against the unforeseeable changes in financial market values that uncertainty breeds.

The erroneous notion of the role of time, as an *ultima ratio* against the prevalence of the ergodic axiom in economics

On several occasions Davidson (1981, p. 61; 1982, p. 16) gave voice to his conviction that the economy is a process moving through historical time. Accordingly, relevant probability distributions are time-dependent, the economic process is nonergodic, and consequently the economic world is not subject to statistical control. The uncertainty highlighted by Keynes and the post-Keynesians met with resistance from numerous other economists because, as Davidson (1988, p. 159) mentions, fundamental uncertainty is treated as an “ill-defined notion which simply muddies the water of scientific investigation,” a notion that is anti-theoretical and ultimately results in nihilism. This is why Dunn (2001) attributes such great significance to Davidson's ergodic/nonergodic dichotomy, because it emphasizes the distinction between deterministic complexity and

fundamental uncertainty. This is borne out by the following opinion quoted from Davidson (1991, p. 133):

“If [...] true uncertainty conditions prevail [...] in certain decision-making areas, then at least some economic processes are such that expectations based on past probability distribution functions can differ persistently from the time averages that will be generated as the future unfolds and becomes historical fact. In these circumstances, sensible economic agents will not rely on available market information regarding relative frequencies, for the future is not statistically calculable from past data and is truly uncertain.”

The outstanding contribution of the post-Keynesians in general – and of Davidson in particular – was their focus on uncertainty; a focus that was missing from classical economics, and is also suspiciously absent from most of the economic literature of today that describes itself as Keynesian. *The post-Keynesians persist with the simple assertion that the future is unknown, and explicitly reject the assumption that past courses of action are destined to repeat themselves in the time to come.*

Time and risk are inherently linked, although without risk appetite the commodity and money-based economic system would wither and fail. Naturally Keynes (1936, p. 161) reminds us that “most, probably, of our decisions to do something positive [...] can only be taken as a result of animal spirits [...] and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities.”

The technique of translating uncertainty into calculable risk rests on a series of conventions. According to Keynes (op. cit. p. 162), one can be described with the assumption that “the existing state of affairs will continue indefinitely, except in so far as we have specific reasons to expect a change [...] We are assuming, in effect, that the existing market valuation, however arrived at, is uniquely correct in relation to our existing knowledge [...] and that it will only change in proportion to changes in this knowledge.” We can pretend to ourselves that the long period is a succession of very short periods, cultivating the illusion that at every moment in time the investor possesses all available information about the future course of share prices. In a particularly subtle passage, Keynes writes that, by using the convention, “an investor can legitimately encourage himself with the idea that the only risk he runs is that of a genuine change in the news over the near future [...] which is unlikely to be very large. [...] Thus investment becomes reasonably ‘safe’ for the individual investor over short periods, and hence over a succession of short periods [...] if he can fairly rely on there being no breakdown in the convention” (op. cit. p. 163).

As Davidson (1993; 1996) has mentioned on several occasions, the pairing of immutable/transmutable reality, which is an ontological distinction, can be matched with the ergodic/nonergodic distinction. We can regard ergodicity as a state where reality cannot be altered, since individuals do not have the power to change outcomes as the drivers of probability distributions are embodied in nature. Nonergodicity, on the other hand, means that reality can be altered, since – in the absence of distributions serving as a solid foundation – it can be changed by a series of forces, including human actions. The consequences are very important from the point of view of rational behaviour, which is eventually determined by the prevailing ontological state of affairs. If reality is ergodic and immutable, then agents will apply probability distributions in their expectations-based decision-making, and such distributions will provide statistically correct outcomes – in time and space. If, however, reality is nonergodic and transmutable, then sensible agents will be forced to act differently, and there are no independent footholds for this behaviour. In the context of investment decisions, under conditions of irreducible uncertainty, mention is made of two related forms of behaviour. In one of these, investors employ an anxiety-free approach, along the lines of “...damn the torpedoes, full speed ahead,” while in the other entrepreneurs create their own future through innovation and new ventures (Davidson, 1982–83, pp. 192–193; 1994, pp. 91, 99; 1996, pp. 482, 497; 2002, pp. 57–58).

A new definition equating ergodicity with immutability (and referring only to the time dimension) redefines the ergodic reality as a situation in which finite time series assembled in the past provide a “reliable estimate” of fundamental distributions, and create a “sample” of future events. Statistically authentic knowledge of unobserved times (and places) can then always be developed based on existing data, thus making it relatively easy to acquire knowledge of the ontological state of affairs.

Based on the approach that calls the ergodic axiom into question, we cannot believe – no matter the available data set – that it is possible to gain a reliable indication of future outcomes. This means that in such an environment, decision-making will be characterized by both a plurality of outcomes and the fact that the dimensions and probability of these outcomes cannot be objectively (mathematically) specified in advance. Accordingly, it would make no sense to objectively assign a given value or probability to the outcome of a given variable. In this regard, it may be emphasized that in such circumstances *uncertainty is associated with a given place and time, and is characterized by unknowable future values and probabilities*. We may draw the conclusion from this that while, on the one hand, the perfect certainty theory of immutability (ergodicity) assumes the availability of total knowledge for the forecasting of future outcomes, those

who espouse the Keynesian theory of economic reality – on the other hand – disagree with this, presupposing that the economy functions in a nonergodic world. Closely related to this idea is the caution from *Kregel* (1976) about the gap between our short-term experiences and our long-term expectations. *A serious problem of the financial market is that in a nonergodic world no relationship necessarily exists between the current behaviour of some variable and the state of long-term expectations with respect to it.*

In discussing the conflict between ergodicity and the notion of time, the polemic between O'Donnell (2014–15) and Davidson (2015) offers some interesting lessons. Davidson (1988, p. 331) previously alluded to the theory that time averages and space (cross-sectional) group averages – with the probability of unity – will converge, while the time averages will coincide with the space averages for infinite realizations. O'Donnell (2014–15) argued that only at infinity can a decision-maker know for certain whether the time statistics and the space statistics will meet, and whether or not the system is ergodic. O'Donnell (op. cit. p. 195) writes the following:

“The convergence/non-convergence that matters occurs at infinity, a never-arriving destination in time and space. With unspecified convergence processes, the pre-infinity signals emitted by reality are unconstrained [...] Furthermore, convergence is only a ‘tendency,’ and tendencies arise when underlying forces are opposed by countervailing forces. Whether the former or the latter currently dominate emitted signals, and over what time and space is unobservable, so that again nothing useful can be inferred from observations of convergence/non-convergence over trivially short spans of time or space. Prior to infinity there can be no necessary or sufficient conditions for ergodicity/nonergodicity.”

O'Donnell (2014–15, p. 192) furthers his argument by asserting that for less than infinity time the path of the statistics over time may actually continually move farther apart even if the system is always ergodic. Of this he writes: “Over finite time and space the two distributions may appear to be (a) converging and hence indicating ergodicity, when in fact they are generated by nonergodicity; or (b) diverging and hence suggesting nonergodicity, when in fact the underlying reality is ergodic.”

Davidson (2015) regards O'Donnell's observations with respect to the behaviour of paths of time and space statistics before infinity as entirely irrelevant. Davidson opines that if the system is ergodic, then at any point of time before infinity the time statistical average and the space (cross-sectional) statistical average may not be equal due to sampling errors. However, if the system is ergodic, the sampling error becomes relatively smaller and the space statistics path and time statistics path should converge, that is, move closer to each other over time.

Here we arrive at the most dubious point in the analogy of ergodicity in the natural sciences, the elimination of the role of time. If there is one thing that renders the applicability of the ergodic hypothesis to (financial) economics fundamentally questionable, it is this. Ergodicity, as a presumption necessary for the existence of equilibrium, enables the individual to formulate statements with respect to a system without having to observe every possible realization of conditions within that system. A single trajectory is sufficient to be able to deduce future behaviour in its entirety, at least on the basis of probability. All this leads to the elimination of time in the scientific description of the economic world. Describing this paradoxical process, Weintraub (1991, p. 102) wrote:

“Equilibrium is then interpreted as the limit of the dynamic behaviour of the system. That is, a solution of the dynamic system involves time, so as time is allowed to pass out of the picture, as it were, or is integrated out by a limiting process, or if we wait until time is no longer meaningful to the statement of the problem.”

Ergodicity thus enables statements to be made about the behaviour of a system as a whole based on a single observed trajectory or – to use an economic term – a single observed time series. In a theoretical approach to ergodic processes, therefore, there is no need to study the history of a process because it is not sensitive to initial conditions; *in an ergodic system time is irrelevant, and has no direction*. No significance is attributed to the elimination of time in equilibrium analysis, when economic equilibrium is interpreted with the help of the equilibrium’s mechanical centre of gravity. Niehans (1997, p. 58) warns of *the dangers of eliminating time*. The consequences are less dramatic where simplification makes analysis significantly easier, although the elimination of time is the price of such simplification. A problem that is not adequately appreciated with respect to ergodicity is that mathematization is often viewed as a victory and an important achievement of science, thereby disregarding the important element of time. It is frequently assumed implicitly – mainly for predictive purposes – that the *entire phase space* is known with respect to future moments in time (this being perfect foresight), or it is assumed that everything remains constant (timeless entity), in the knowledge that neither assumption has any scientific or empirical basis. The environment of economic processes is real time, with everything occurring in *historical time*, and in such a medium the supposed causal relationship embodied in ergodicity is false and misleading because *the causal relationship has no relevance*.

Set against our broad-reaching attempts at rebuttal, a number of stochastic theorists have taken the view that ergodic theory is necessary to forecast future outcomes. For example, Billingsley (1978, p. 1) states that if “the laws governing change remain fixed as time passes, [then] ergodic theory is a key to understanding these fluctuations.” Billingsley (1978, p. 2) goes on to state that

whenever the “passage of time does not affect the set of joint probability laws governing experimentation (outcomes), then the assumption of ergodicity permits regularities to be perceived from what might at first sight be patternless fluctuations.” In other words, assuming ergodicity permits a past pattern of regularities to be reliably projected into the future.

Keynes had fundamental doubts about the potential relevance of transplanting processes directly from physics to economics. He refused to support this potential transfer even despite Ehrenfest’s backing of the applicability of physical categories to economics. Tinbergen (1939) also supported the existence of a stable natural law underpinning economic time series – despite the fact that he was also sceptical that such series are a reflection of current equilibrium outcomes. *We regard both the epistemological and ontological aspects of uncertainty as valid, both supplying an argument to defenders of the idea of free markets in declaring that no economic actors can know what the future will bring.* Accordingly, it is best to leave individuals to themselves to make their own decisions and commit their own errors on the market, thus resulting in the survival of the fittest.

Shackle (1958) not only expressed doubt with respect to the validity of formal, mechanical time dynamics models, but also excluded any type of forward-looking economic model, except for that directed at the next period. In his view, predictions could be made for one period ahead, given the state of current expectations and intentions. As the next period runs out, market participants form new expectations, not necessarily connected to something that has already happened earlier, and it is possible that their actions are based on something new in the market (for example, potentially involving a real investment in innovation).

Time is a medium that surrounds and envelopes economic processes, where occurrences within these processes, and the uncertainty thereof, are linked to points or periods in time, and cannot be separated from time itself. If the world full of decision-makers were ergodic, then any uniformity attributed to history would be misleading. We thus regard assumptions about the relevance of ergodicity as false because we have doubts over the timelessness and immutability of economic processes. Davidson (1988, p. 332) warned that if relationships between economic variables are ergodic, then time will not appear as an essential factor. The presumption of an immutable system in which history is predetermined, stochastically or otherwise, as Shackle (1972) and Davidson (1996) repeatedly stated, sharply contradicts any sense of rational choice. In so far as the economic world is ergodic, outcomes in the long run are pre-programmed and independent of decisions taken by economic participants. Choice is not natural, unimportant, and – in the longer term – does not even matter. Any attempt made to assign uniformity and regularity to human actors in history is misleading.

REFERENCES

- ALVAREZ, M. C. – EHNTS, D. H. (2012): *The Theory of Reflexivity – A Non-Stochastic Randomness Theory for Business Schools Only?* IPE Working Papers 28/2012.
- ALVAREZ, M. C. – EHNTS, D. H. (2012): *Samuelson and Davidson on Ergodicity: A Reformulation.* *Journal of Post Keynesian Economics*, Vol. 39, No. 1, 1–16.
- BACHELIER, L. (1900): Theory of Speculation. In: COOTNER, P. H. (ed.): *The Random Character of Stock Market Prices.* MIT Press, Cambridge, 1964, pp. 17–78.
- BERNSTEIN, P. L. (1998): Stock Market Risk in a Post Keynesian World. *Journal of Post Keynesian Economics* (Fall), Vol. 21, No. 1.
- BILLINGSLEY, P. (1978): *Ergodic Theory and Information.* Huntington, NY: Krieger.
- BIRKHOFF, G. (1931): *Proof of the Ergodic Theorem.* Proceedings of the National Academy of Sciences, Vol. 17, pp. 656–660.
- BOLTZMANN, L. (1884): Über die Eigenschaften Monozyklischer und anderer damit Verwandter Systeme. *Journal für die reine und angewandte Mathematik. (Journal de Crelle)*, 98, pp. 68–94.)
- BORTKIEWICZ, L. (1918): Homogeneity and Stability in the Statistics. *Skandinavisk Aktuarietidskrift*, 1–2, pp. 1–81.
- BRONK, R. (2009): *The Romantic Economist – Imagination in Economics.* Cambridge University Press.
- BRONK, R. (2011): *Epistemological Difficulties with Neoclassical Economics.* The London School of Economics and Political Science, pp. 1–22.
- BUCHANAN, J. M. – VANBERG, V. J. (1991): The Market as a Creative Process. *Economics and Philosophy*, Vol. 7.
- CLAUSIUS, R. (1857): The Nature of the Motion which we Call Heat (English translation). *Philosophical Magazine*, Vol. 14, pp. 108–127.
- CODDINGTON, A. (1982): Deficient Foresight: A Troublesome Theme in Keynesian Economics. *American Economic Review*, Vol. 72, pp. 480–487.
- COOTNER, P. H. (1962): Stock Prices: Random versus Systematic Changes. *Industrial Management Review*, Vol. 3, No. 2, p. 24.
- COWLES, A. (1933): Can Stock Market Forecasters Forecast? *Econometrica*, Vol. 1, No. 3, pp. 309–324.
- DAVIDSON, P. – DAVIDSON, G. S. (1984): Financial Markets and Williamson's Theory of Governance: Efficiency vs. Concentration vs. Power. Reprinted in: *Money and Employment: The Collected Writings of Paul Davidson* (1991), Vol. I, London: Macmillan.
- DAVIDSON, P. (1972): *Money and the Real World.* London: Macmillan.
- DAVIDSON, P. (1978): *Money and the Real World.* Second edition, London: Macmillan.
- DAVIDSON, P. (1982–83): Rational Expectations: A Fallacious Foundation for Studying Crucial Decision-Making Processes. *Journal of Post Keynesian Economics*, Vol. 5, No. 2, pp. 182–197.
- DAVIDSON, P. (1988): A Technical Definition of Uncertainty and the Long-Run Non-Neutrality of Money. *Cambridge Journal of Economics*, Vol. 12, pp. 329–337.
- DAVIDSON, P. (1989): The Economics of Ignorance or Ignorance of Economics? *Critical Review*, Vol. 3, Nos. 3–4, pp. 467–487.
- DAVIDSON, P. (1991): Is Probability Theory Relevant for Uncertainty? A Post-Keynesian Perspective. *Journal of Economic Perspectives*, Vol. 5, No. 1, pp. 129–143.

- DAVIDSON, P. (1993): Austrians and Post Keynesians on Economic Reality: A Rejoinder to Critics. *Critical Review*, Vol. 7.
- DAVIDSON, P. (1994): *Post Keynesian Macroeconomic Theory*. Edward Elgar, Aldershot, United Kingdom.
- DAVIDSON, P. (1996): Reality and Economic Theory. *Journal of Post Keynesian Economics*, Vol. 18, No. 4, pp. 479–508.
- DAVIDSON, P. (2002): *Financial Markets, Money and the Real World*. Elgar, Cheltenham, UK.
- DAVIDSON, P. (2006): *Samuelson and the Keynes/Post Keynesian Revolution*. New School Working Paper.
- DAVIDSON, P. (2009): Can Future Systemic Financial Risks be Quantified? Ergodic versus Nonergodic Stochastic Processes. *Brazilian Journal of Political Economy*, Vol. 29, No. 4, pp. 324–340.
- DAVIDSON, P. (2010): Black Swans and Knight's Epistemological Uncertainty. *Journal of Post Keynesian Economics*, Vol. 4, pp. 567–570.
- DAVIDSON, P. (2012): Is Economics a Science? Should Economics be Rigorous? *Real-World Economics Review*, March.
- DAVIDSON, P. (2015): A Rejoinder to O'Donnell's Critique of the Ergodic/Nonergodic Explanation of Keynes's Concept of Uncertainty. *Journal of Post Keynesian Economics*, Vol. 38, pp. 1–18.
- DEQUECH, D. (2000): Fundamental Uncertainty and Ambiguity. *Eastern Economic Journal*, Vol. 26, No. 1, pp. 41–60.
- DHRYMES, P. (1974): *Econometrics: Statistical Foundations and Applications*. New York: Springer-Verlag.
- DUNN, S. P. (2000): Fundamental Uncertainty and the Firm in the Long Run. *Review of Political Economy*, Vol. 12, No. 4, pp. 419–433.
- DUNN, S. P. (2001): Bounded Rationality is Not Fundamental Uncertainty: A Post Keynesian Perspective. *Journal of Post Keynesian Economics*, Summer, Vol. 23, pp. 567–583.
- DUNN, S. P. (2012): "Non-Ergodicity." In: KING, J. E. (ed.): *The Elgar Companion to Post Keynesian Economics*. Cheltenham UK and Northampton, MA: Edward Elgar, pp. 434–439.
- EGERTON, R. A. D. (1955): Investment, Uncertainty and Expectations. *The Review of Economic Studies*, Vol. 58, pp. 143–150.
- EHRENFEST, P. – EHRENFEST-AFANESSJEW, T. (1912): Begriffliche Grundlagen der statistischen Auffassung in der Mechanik. In: KLEN, F. – MÜLLER, C. (eds.): *Encyklopädie der Mathematischen Wissenschaften*, Vol. 4, Leipzig: Teubner, pp. 3–90.
- EHRENFEST, P. (1911): *Begriffliche Grundlagen der statistischen Auffassung in der Mechanik*. Leipzig: B. G. Teubner.
- ENGLER, R. – GRANGER, C. (1987): Co-Integration and Error Correction: Representation, Estimation and Testing. *Econometrica*, Vol. 55, pp. 251–276.
- FITZGIBBONS, A. (1996): The Logic of Post Keynesian Economics. *History of Economics Review*, Winter, pp. 72–77.
- FORD, J. L. (1993): G. L. S. Shackle (1903–1992): A Life with Uncertainty. *The Economic Journal*, May, Vol. 103, pp. 683–697.
- FRIEDMAN, M. (1953): *Essays in Positive Economics*. Chicago: University of Chicago Press.
- GALLAVOTTI, G. (1995): Ergodicity, Ensembles, Irreversibility in Boltzmann and Beyond. *Journal of Statistical Physics*, Vol. 78, pp. 1571–1589.
- GALLAVOTTI, G. (1999): *Statistical Mechanics: A Short Treatise*. Berlin: Springer-Verlag.

- GIBBS, J. W. (1902): *Elementary Principles in Statistical Mechanics. Developed with Special Reference to the Rational Foundation of Thermodynamics*. Charles Scribner's Sons, New York.
- GILLIES, P. (2003): Probability and Uncertainty in Keynes's *The General Theory*. In: RUNDE, J. – MIZUHARA, S. (eds.): *The Philosophy of Keynes's Economics: Probability, Uncertainty and Convention*. London: Routledge.
- HALMOS, P. R. (1949): Measurable Transformations. *Bulletin of the American Mathematical Society*, November, pp. 1015–1034.
- HALMOS, P. R. (1958): Von Neumann on Measure and Ergodic Theory. *Bulletin of the American Mathematical Society*, Vol. 64, pp. 86–94.
- HAUSMANN, D. M. (1994): *The Philosophy of Economics: An Anthology*, 2nd edition, Cambridge University Press.
- HAZEWINKEL, M. [ed.] (2002): Ergodic Theory. In: *Encyclopaedia of Mathematics*, New York: Springer-Verlag/SpringerLink.
- HICKS, J. R. (1931): The Theory of Uncertainty and Profit. *Economica*, Vol. 11, No. 32, pp. 170–189.
- HICKS, J. R. (1939): *Value and Capital*. Clarendon Press, Oxford.
- HICKS, J. R. (1979): *Causality in Economics*. Basic Books, New York.
- HODGSON, G. (2011): The Eclipse of the Uncertainty Concept in Mainstream Economics. *Journal of Economic Issues*, Vol. 45, No. 1, pp. 159–175.
- JOVANOVIĆ, F. – SCHINCKUS, C. (2012): *The History of Econophysics' Emergence: A New Approach in Modern Financial Theory*. University of Quebec at Montreal, pp. 1–46.
- JOVANOVIĆ, F. (2001): Pourquoi l'hypothèse de marche aléatoire en théorie financière? Les raisons historiques d'un choix éthique. *Revue d'Economie Financière*, Vol. 61, pp. 203–211.
- JOVANOVIĆ, F. (2008): The Construction of the Canonical History of Financial Economics. *History of Political Economy*, Vol. 40, No. 2, pp. 213–242.
- KENDALL, M. G. (1953): The Analysis of Economic Time Series. Part I: Prices. *Journal of the Royal Statistical Society*, Vol. 116, pp. 11–25.
- KEYNES, J. M. (1907): *The Principles of Probability*. Fellowship Dissertation, King's College, Cambridge University, December.
- KEYNES, J. M. (1921): *A Treatise on Probability*. London: Macmillan.
- KEYNES, J. M. (1936): *The General Theory of Employment, Interest and Money*. New York: Harcourt, Brace and Company.
- KEYNES, J. M. (1937): The General Theory of Employment. *The Quarterly Journal of Economics*, Vol. 51, No. 2, pp. 209–223.
- KEYNES, J. M. (1939): Official Papers. The League of Nations. Professor Tinbergen's Method. *The Economic Journal*, Vol. 49, No. 195, pp. 558–568.
- KIRSTEIN, M. (2015): *From the Ergodic Hypothesis in Physics to the Ergodic Axiom in Economics*. Prepared for the 7. Wintertagung des ICAE, Linz.
- KNIGHT, F. H. (1921): *Risk, Uncertainty and Profit*. Boston, MA: Hart, Shaffner and Marx; Houghton Mifflin Co.
- KREGEL, J. A. (1976): Economic Methodology in the Face of Uncertainty: The Modelling Methods of Keynes and the Post-Keynesians. *Economic Journal*, Vol. 86, pp. 209–225.
- KREGEL, J. A. (1998): Aspects of a Post Keynesian Theory of Finance. *Journal of Post Keynesian Economics*, Vol. 21, No. 1, pp. 111–133.
- LAPLACE, P. S. (1812): *Théorie analytique des probabilités*. Paris: Gauthier-Villars.
- LAWSON, T. (1985): Uncertainty and Economic Analysis. *Economic Journal*, Vol. 95, pp. 909–927.

- LAWSON, T. (1988): Probability and Uncertainty in Economic Analysis. *Journal of Post Keynesian Economics*, Vol. 11, No. 1, pp. 38–65.
- LEXIS, W. (1875): *Einsetzung in der Theoretischer Bevölkerungsstatistik*. Strasbourg: Karl Trübner.
- LI, D. X. (2000): On Default Correlation: A Copula Function Approach. *Journal of Fixed Income*, Vol. 9, No. 4, pp. 43–54.
- LUCAS, R. E. – SARGENT, T. J. (1981): *Rational Expectations and Econometric Practice*. Minneapolis: University of Minnesota Press.
- LUCAS, R. E. (1981): *Studies in Business Cycle Theory*. Cambridge, MA: MIT Press.
- MACKEY, G. (1974): Ergodic Theory and its Significance for Statistical Mechanics and Probability Theory. *Advances in Mathematics*, Vol. 12, pp. 178–268.
- MALINVAUD, E. (1949): *Statistical Methods of Econometrics*. Amsterdam: North Holland.
- MARSHALL, A. (1920): *Principles of Economics*. 8th ed., London: Macmillan and Co.
- MAXWELL, J. C. (1860): Illustrations of the Dynamical Theory of Gases. *Philosophical Magazine*, Vol. 19, pp. 19–32.
- MAXWELL, J. C. (1867): On the Dynamical Theory of Gases. *Philosophical Transactions of the Royal Society*, Vol. 157, pp. 49–88.
- MCCAULEY, J. (2004): *Dynamics of Markets: Econophysics and Finance*. Cambridge UK: Cambridge University Press.
- MIROWSKI, P. (1989): The Probabilistic Counter-Revolution, or How Stochastic Concepts Came to Neoclassical Economic Theory. *Oxford Economic Papers*, Vol. 41, pp. 217–235.
- NEUMANN, J. V. (1932): Proof of the Quasi-Ergodic Hypothesis. *Proceedings of the National Academy of Sciences*, Vol. 18, pp. 255–263.
- NIEHANS, J. (1959): Reflections on Shackle, Probability and our Uncertainty about Uncertainty. *Metroeconomica*, Vol. 11, pp. 74–78.
- NIEHANS, J. (1997): *Samuelsons "Foundations": Ein nichtmathematischer Leitfaden*. Düsseldorf: Verlag Wirtschaft und Finanzen, pp. 55–87.
- O'DONNELL, R. (1990): An Overview of Probability, Expectations, Uncertainty and Rationality in Keynes's Conceptual Framework. *Review of Political Economy*, Vol. 2, No. 3, pp. 253–256.
- O'DONNELL, R. (2013): Two Post-Keynesian Approaches to Uncertainty and Irreducible Uncertainty. In: HARCOURT, G. C. and KRIESLER, P. (eds.): *The Oxford Handbook of Post-Keynesian Economics. Vol. 2: Critiques and Methodology*. Oxford University Press.
- O'DONNELL, R. (2014–15): A Critique of the Ergodic/Nonergodic Approach to Uncertainty. *Journal of Post Keynesian Economics*, Vol. 37, No. 2, pp. 187–209.
- PETERS, O. (2009): On Time and Risk. *Santa Fe Institute Bulletin*, pp. 36–41.
- POINCARÉ, H. (1893): Le Mécanisme et l'Expérience. *Revue de Métaphysique et de Morale*, Vol. 1, pp. 534–537.
- POITRAS, G. (2012): *Ergodicity, Econophysics and Financial Economics*. San Francisco, CA, June, pp. 1–42.
- REGNAULT, J. (1863): *Calcul des chances et philosophie de la bourse*. Paris: Mallet-Bachelier et Castel.
- ROSSER, J. B., Jr. (1999): On the Complexities of Complex Economic Dynamics. *Journal of Economic Perspectives*, Vol. 13, No. 3, pp. 169–192.
- ROSSER, J. B., Jr. (2001): Alternative Keynesian and Post Keynesian Perspectives on Uncertainty and Expectations. *Journal of Post Keynesian Economics*, Vol. 23, No. 4, pp. 545–566.
- ROSSER, J. B., Jr. (2003): *Complexity in Economics*. New York: Edward Elgar.

- ROSSER, J. B., JR. (2007): *Econophysics and Economic Complexity*. Working Paper.
- ROSSER, J. B., JR. (2015): Reconsidering Ergodicity and Fundamental Uncertainty. James Madison University, July, pp. 1–27.
- RUNDE, J. (1990): Keynesian Uncertainty and the Weight of Arguments. *Economics and Philosophy*, Vol. 6, pp. 275–292.
- SAMUELSON, P. A. (1947): *Foundations of Economic Analysis*. Boston, MA: Harvard University Press.
- SAMUELSON, P. A. (1948): *Economics: An Introductory Analysis*. New York: McGraw-Hill.
- SAMUELSON, P. A. (1965): Proof that Properly Anticipated Prices Fluctuate Randomly. *Industrial Management Review*, Vol. 6, No. 2, pp. 41–69.
- SAMUELSON, P. A. (1965): Rational Theory of Warrant Pricing. *Industrial Management Review*, Vol. 6, No. 2, pp. 13–40.
- SAMUELSON, P. A. (1968): What Classical and Neoclassical Monetary Theory Really Was. *Canadian Journal of Economics*, Vol. 1, No. 1, pp. 1–15.
- SAMUELSON, P. A. (1969): Classical and Neoclassical Theory. In: CLOWER, R. W. (ed.): *Monetary Theory: Selected Readings*. Harmondsworth: Penguin Modern Economics Readings, pp. 182–194.
- SAMUELSON, P. A. (1976): Optimality of Sluggish Predictors under Ergodic Probabilities. *International Economic Review*, Vol. 17, pp. 1–7.
- SAMUELSON, P. A. (1998): How Foundations Came to Be. *Journal of Economic Literature*, Vol. 36, No. 3, pp. 1375–1386.
- SAUTER, O. (2014): *Monetary Policy under Uncertainty: Historical Origins, Theoretical Foundations, and Empirical Evidence*. Wiesbaden: Springer Gabler.
- SHACKLE, G. L. S. (1949): *Expectation in Economics*. Cambridge: Cambridge University Press.
- SHACKLE, G. L. S. (1949–50): A Non-Additive Measure of Uncertainty. *Review of Economic Studies*, Vol. 17, pp. 70–74.
- SHACKLE, G. L. S. (1953): The Logic of Surprise. *Economica*, Vol. 20, No. 78, pp. 112–117.
- SHACKLE, G. L. S. (1955): *Uncertainty in Economics and Other Reflections*. Cambridge: Cambridge University Press.
- SHACKLE, G. L. S. (1958): Expectation and Liquidity. In: BOWMAN, M. J. (ed.): *Expectations, Uncertainty and Business Behavior*. New York: Social Science Research Council.
- SHACKLE, G. L. S. (1961): *Decision, Order and Time in Human Affairs*. Cambridge: Cambridge University Press.
- SHACKLE, G. L. S. (1972): *Epistemics and Economics*. Cambridge: Cambridge University Press.
- SHACKLE, G. L. S. (1974): *Keynesian Kaleidics: The Evaluation of a General Political Economy*. Edinburgh: Edinburgh University Press.
- SHACKLE, G. L. S. (1980): Imagination, Unknowledge and Choice. *Greek Economic Review*, Vol. 2, pp. 95–100.
- SOLOW, R. M. (1985): Economic History and Economics. Papers and Proceedings, *American Economic Review*, Vol. 75, pp. 328–331.
- SYLL, L. P. (2012): Uncertainty and Ergodicity – The Important Difference between Keynes and Knight. Discussion with Paul Davidson. *Real-World Economics Review Blog*, March 30.
- THEIL, H. (1971): *Principles of Econometrics*. New York: Wiley.
- TINBERGEN, J. (1937): *An Econometric Approach to Business Cycle Problems*. Paris: Hermann.
- TINBERGEN, J. (1939): *A Method and its Application to Investment Activity*. Geneva: League of Nations, pp. 1–169.

- TURVEY, R. (1949): Three Notes on “Expectation in Economics,” Part III. *Economica*, Vol. 16, pp. 336–338.
- UFFINK, J. (2006): *Compendium of the Foundations of Classical Statistical Physics*. Institute for History and Foundations of Science, University of Utrecht.
- WALD, H. O. A. (1965): *Bibliography on Time Series and Stochastic Processes*. Cambridge Mass: MIT Press.
- WEINTRAUB, E. R. (1991): *Stabilizing Dynamics: Constructing Economic Knowledge*. Cambridge, MA: Cambridge University Press.
- WEINTRAUB, E. R. (2002): *How Economics Became a Mathematical Science*. Durham, NC: Duke University Press.
- WIENER, N. (1939): The Ergodic Theorem. *Duke Mathematical Journal*, Vol. 5, No. 1, pp. 1–18.
- WORDSWORTH, W. (1798): Lines Composed a Few Miles above Tintern Abbey. Reprinted in Wu, D. [ed.] (1998): *Romanticism: An Anthology*. 2nd edition, Blackwell, pp. 265–269.
- WORKING, H. (1934): A Random-Difference Series for Use in the Analysis of Time Series. *Journal of the American Statistical Association*, Vol. 29, pp. 11–24.
- ZAPPÀ, C. – BASILI, M. (2005): *Shackle versus Savage: Non-Probabilistic Alternatives to Subjective Probability Theory in the 1950s*. Università degli Studi di Siena, No. 452, pp. 1–33.